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Direction of Arrival and Time-Delay Estimation with an ESPAR
Antenna Employing the SAGE Algorithm

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Abstract

The electronically steerable parasitic array radiator (ESPAR) antenna is a low complexity adaptive antenna, which accomplishes the beamforming in the analog domain. The sequential SAGE algorithm aims at the maximum-likelihood estimate for the unknown parameter set, though convergence is not guaranteed. In this paper we investigate a direction-of-arrival (DoA) and delay estimation technique for an ESPAR antenna, employing the space-alternating generalized expectation-maximization (SAGE) algorithm. The results from computer simulations reveal that the DoAs of two sinusoidal signals can be resolved with a high accuracy, whereas the DoAs and delays of a large number of signals can be found if the signal waveforms exhibit pseudo-noise sequence characteristics.

Key words: ESPAR antenna, DoA and delay estimation, SAGE algorithm, maximum likelihood estimation.

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Glossary and Notations

CDMA: Code Division Multiple Access

DoA: Direction of Arrival

EM: Expectation Maximization

ESPAR: Electronically Steerable Parasitic Array Radiator

ESPRIT: Estimation of Signal Parameters via Rotational Invariance Techniques

IEEE: Institute of Electrical and Electronics Engineers

IEICE: Institute of Electronics, Information and Communication Engineers

ML: Maximum Likelihood

MUSIC: Multiple Signal Classification

PN-sequences: Pseudo Noise sequences

SAGE: Space-Alternating Generalized Expectation-maximization

SNR: Signal to Noise Ratio

WGN: White Gaussian Noise

To characterize all the mathematical variable in this report, we used some notations, as shown below,

- A scalar or complex value is noted: x
- A vector is noted: \mathbf{x}
- A matrix is noted: \mathbf{X}
- And a constant is noted: D

Introduction

Due to evolution of the communication system and all the new services, it will be necessary to find other technical solutions to give to the clients a good quality of service. But updating the present network architecture is more and more complex. In the last few years a new network concept, called Wireless Ad-hoc Community Network (WACNet), was introduced by the Advanced Telecommunications Research (ATR) institute. The WACNet concept aims to form an ad-hoc community which are formed with an unspecified number of terminals temporally connected with a common purpose.

New concept imply new technics therefore a specific antenna is proposed, Electronically Steerable Parasitic Array Radiator (ESPAR) antenna. An ESPAR antenna is a reactively controlled directive array that enables full azimuth directivity. The main difference with a conventional antenna is the number of output port, as the ESPAR antenna is a single output port. This antenna has low cost, low power consumption and high power level. The department 3 of the Adaptive Communication Research (ACR) labs of ATR is in charge of studying and improving this antenna by applying different type of adaptive algorithm.

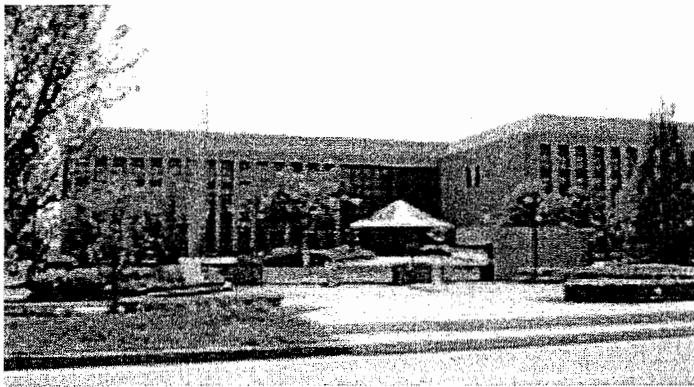
The determination of the parameters of multiple wave-fronts impinging on an array antenna is a significant issue in digital array processing. In this report, the purpose is to apply Space-Alternating Generalized Expectation-maximization (SAGE) algorithm for the ESPAR antenna. This algorithm aims to find the parameters of multiple signals arriving on the ESPAR antenna. The results from computer simulations reveal that the parameters signals can be resolved with a high accuracy.

After presenting ATR and ACR labs, the ESPAR antenna will be introduced. Next, the purpose and background of the project will be presented. Then, we study the efficiency of the SAGE algorithm with the Cramer-Rao bound.

Chapter 1

ATR - Advance Telecommunication Research

1.1 Introduction to ATR



ATR is an independent corporation which conducts Research and Development (R&D), both basic and advanced, in the field of telecommunications. Research is commissioned by public and private sectors, or carried out with funds obtained through open competition. We are also actively engaged in applying our research results to industrial and commercial use.

In the information society of the 21st century, great volumes of diverse information will be disseminated through various media. This vast, varied information exchange will require advanced systems for accurate and efficient processing of information. Such systems must be easy to handle or operate. They must also be freely accessible and readily integrated into our day-to-day lives. These are vital aspects of this important research:

- New multi-media communication systems
- Cross-language global communication systems
- Ideal human-machine interfaces
- Developing fundamental technologies for highly adaptive communication systems

The ATR Group was established in March 1986 with support from various sections of industry, academia and government to serve as a major center of basic and creative telecommunications R&D. Its basic policy is to promote research collaboration among researchers both at home and abroad. The ATR Group comprises nine corporations: R&D corporations, four corporations designed to manage research fruits, and an umbrella corporation, ATR International. While the former four are now conducting research, the latter four have completed their R&D programs and are now managing the fruits of their research to promote their propagation and wider application. The four R&D corporations' activities are funded by the Japan Key Technology Center (70%) and 136 private companies (30%).

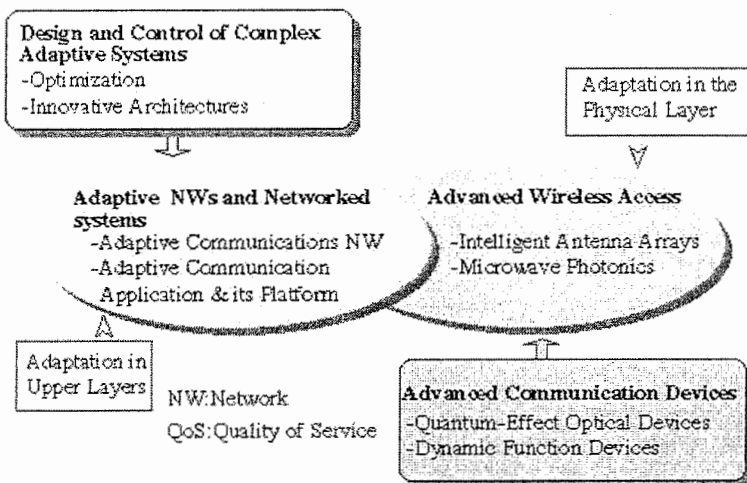
CAPITAL: ATR International : 22.0325 billion YEN (Invested by 136 Companies)

1.2 ACR Laboratories

1.2.1 Research domains

Focusing on wireless technologies, we are conducting research on fundamental technologies for user-friendly communications systems where users need not concern themselves with the complexities of how networks work. Currently, we are emphasizing key technologies for wireless ad hoc networks, consisting of mobile nodes which do not need the assistance of centralized infrastructure such as base stations.

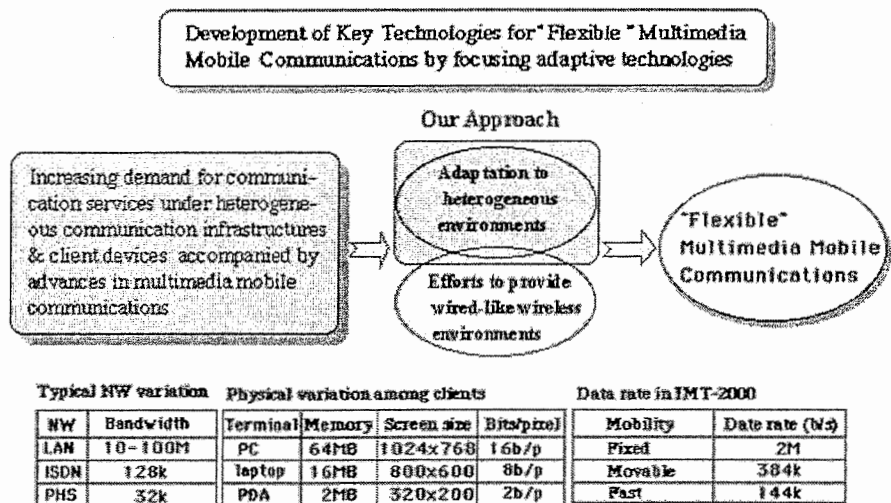
-Adaptive Technologies for Multimedia Wireless Communications-



1.2.2 ACR Department 3

Research is being carried out on smart antennas, promising key components for future wireless ad-hoc networks. The department 3 is developing an ESPAR (Electronically steerable parasitic array radiator) antenna as the smart antenna. Since the ESPAR antenna steers autonomously its antenna beam toward the arrival direction radio waves and steers the nulls of the beam toward the undesired interfering waves, power-savings and frequency reusability in ad-hoc networks can be greatly improved. Moreover, lots of experiments are made in the compact radio anechoic box. Conventionally, a large and expensive room-size radio anechoic chamber has been required for the measurement evaluation and calibration of antennas because electromagnetic fields have to be measured at a distance from an antenna. Utilizing advanced microwave photonics-based sensors, we are aiming at developing a compact radio anechoic box that can pick up electromagnetic fields in close proximity to an antenna without external disturbance.

1.2.3 Main purpose project



Chapter 2

Introduction to the ESPAR Antenna

In this section, we briefly describe the basic configuration and give the formulation of the ESPAR antenna.

2.1 Configuration of the ESPAR Antenna

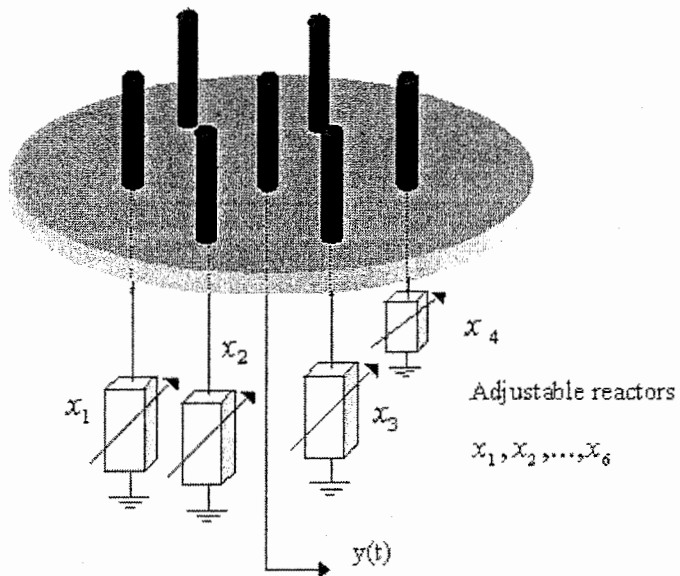


Figure 2.1: Diagram of an adaptive ESPAR antenna.

An M -element ESPAR antenna has one active and $M - 1$ passive

antenna elements. Typically, $M=7$ and the six passive elements surround the active element with a radius of a quarter of the wavelength. It is a $\lambda/4$ -monopole where λ denote the wavelength and is excited from the bottom in a coaxial fashion. The pattern of this antenna is controlled by adjusting the values x_m ($m = 1, \dots, M-1$) of the reactances connected to the parasitic radiators as shown in Fig. 2.1. In practical applications, the reactance x_m may be constrained in certain ranges, e.g., from -300Ω to 300Ω .

2.2 ESPAR antenna Formulation

Consider the $M+1$ -element ESPAR antenna [1] as shown in Fig. 2.2. The m -th element is placed at an angle $\phi_m = \frac{2\pi}{M}(m-1)$, ($m = 1, 2, \dots, M$) relative to an arbitrary axis. When an incoming wavefront is impinging on the antenna from a Direction-of-Arrival (DoA) of θ relative to the same reference axis, there is a spatial delay of $R\cos(\theta - \phi_m)$ between the signals received at the pair of the m -th element and the 0-th element. Where R is the radius of the antenna. This spatial delay can be expressed as an electrical angular difference defined by $\frac{2\pi}{M}R\cos(\theta - \phi_m)$. Then the steering vector of the ESPAR antenna on the DoA of θ is, when radius is $R = \frac{\lambda}{4}$, defined by

$$\mathbf{a}(\theta) = (a_1(\theta), \dots, a_m(\theta), \dots, a_M(\theta))^T,$$

with

$$a_m(\theta) = \exp\left(-j2\pi\frac{|Z_m|}{\lambda}\cos(\text{ang}(Z_m) - \theta)\right),$$

where the complex-valued Z_m defining the position of the m -th passive element on the antenna plane and λ the wavelength of the radio frequency (RF) signals. The subscript T is the transpose operation.

Suppose there are D arriving signals $u_d(t)$ with the DoAs θ_d ($d=1, 2, \dots, D$). Let $\mathbf{s}_m(t)$ denote the composite signal impinging at the m -th antenna element and let $\mathbf{s}(t)$ be the vector signal

$$\mathbf{s}(t) = (s_1(t), \dots, s_M(t))^T.$$

Then, $\mathbf{s}(t)$ may be expressed as

$$\mathbf{s}(t) = \sum_{d=1}^D \mathbf{a}(\theta_d)u_d(t),$$

where $\mathbf{a}(\theta_d)$ is the steering vector depending on the structure of the ESPAR antenna.

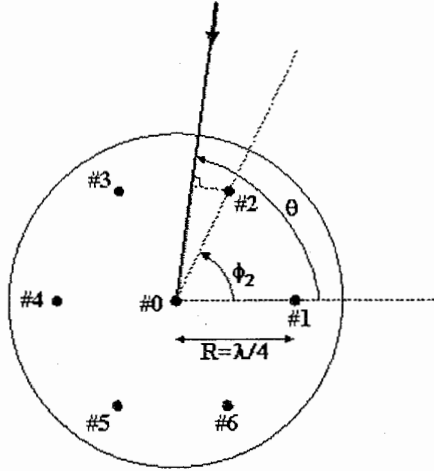


Figure 2.2: Geometry of an ESPAR antenna.

Then, the signal at the ESPAR antenna output port can be written as [1, 2, 3]

$$y(t) = \mathbf{i}^T \mathbf{s}(t) + v(t),$$

where $v(t)$ is an additive white Gaussian noise (WGN) signal, and \mathbf{i} is the RF current vector. We defined also, the RF voltage vector

$$\mathbf{v} = V_s \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \mathbf{X}\mathbf{i}, \quad (2.1)$$

where the matrix $\mathbf{X} = \text{diag}(50, jx_1, jx_2, \dots, jx_M)$ is a diagonal matrix called the reactance matrix and the vector $\mathbf{i} = (i_1(t), \dots, i_M(t))^T$. Additionally, V_s is the internal RF source voltage of the active radiator. And the relationship between the RF voltage and RF current vector is

$$\mathbf{i} = \mathbf{Y}\mathbf{v}, \quad (2.2)$$

where \mathbf{Y} is the $(M + 1) \times (M + 1)$ admittance matrix, with

$$\mathbf{Y} = \begin{bmatrix} y_{00} & \cdots & y_{0M} \\ \vdots & \ddots & \vdots \\ y_{M0} & \cdots & y_{MM} \end{bmatrix}$$

and y_{kl} represents the mutual admittance between the elements k and l for $(k, l) \in [0, M]^2$. Substituting equation (2.1) into equation (2.2) yields

$$\mathbf{i} = V_s(\mathbf{I} + \mathbf{Y}\mathbf{X})^{-1}\mathbf{y}_0, \quad (2.3)$$

where \mathbf{I} is the identity matrix of order $(M \times 1)$ and \mathbf{y}_0 the first column of \mathbf{Y} . Furthermore, by the reciprocity theorem, it holds, similarly to conventional array antennas, that

$$\forall(k, l) \in [0, M]^2, \quad y_{kl} = y_{lk}. \quad (2.4)$$

In addition, the cyclic symmetry of the elements of the ESPAR antenna implies

$$\begin{aligned} y_{11} &= y_{22} = y_{33} = y_{44} = y_{55} = y_{66} \\ y_{01} &= y_{02} = y_{03} = y_{04} = y_{05} = y_{06} \\ y_{12} &= y_{23} = y_{34} = y_{45} = y_{56} = y_{61} \\ y_{13} &= y_{24} = y_{35} = y_{46} = y_{51} = y_{62} \\ y_{14} &= y_{25} = y_{36} \end{aligned} \quad (2.5)$$

Finally, equation (2.4) and equation (2.5) imply that the admittance matrix \mathbf{Y} is determined by only 6 components of the mutual admittances y_{00} , y_{10} , y_{11} , y_{21} , y_{31} and y_{41} . The values of these 6 components depend on the physical structure of the antenna, *e.g.*, the radius, the space intervals and the lengths of the elements, and therefore are constant.

Summarizing the above explanation, we write the admittance matrix \mathbf{Y} as

$$\mathbf{Y} = \begin{bmatrix} y_{00} & y_{10} & & & & & & y_{10} \\ y_{10} & y_{11} & y_{21} & y_{31} & y_{41} & y_{31} & y_{21} & \\ & & y_{21} & \ddots & \ddots & \ddots & \ddots & y_{31} \\ \vdots & & & y_{31} & \ddots & \ddots & \ddots & y_{41} \\ & & & & y_{41} & \ddots & \ddots & y_{31} \\ & & & & & y_{31} & \ddots & y_{21} \\ y_{10} & y_{21} & y_{31} & y_{41} & y_{31} & y_{21} & y_{11} & \end{bmatrix}.$$

Similarly, the \mathbf{y}_0 vector can be rewritten as $\mathbf{y}_0 = [y_{00}, y_{10}, y_{10}, \dots, y_{10}]^T$. The \mathbf{i} current vector can also be formulated as

$$\mathbf{i} = V_s(\mathbf{Z} + \mathbf{X})^{-1}\mathbf{u}_0,$$

where \mathbf{u}_0 is the $(M+1) \times 1$ vector $(1, 0, \dots, 0)^T$. The matrix \mathbf{Z} components are constants and determined by the mutual admittances between the elements. We may express the ESPAR output $y(t)$ also as

$$y(t) = \sum_{d=1}^D \mathbf{i}^T \mathbf{a}(\theta_d) \alpha_d x_d(t - \tau_d) + v(t),$$

here, $x_d(t)$ denotes the original d -th signal and $\alpha_d, \theta_d, \tau_d$ are its complex-valued amplitude, impact angle, and delay, respectively. The signals $x_1(t), \dots, x_D(t)$ are assumed periodical, i.e., $x_d(t) = x_d(t + T_S) \forall t \in \mathbb{R}$ with T_S being a constant, and originating from either a common source or from different transmitters. The DoA and delay estimation is accomplished on the basis of the ESPAR antenna output for N different antenna patterns. That is, we choose N different reactance settings, i.e., N different current vectors $\mathbf{i}_1, \dots, \mathbf{i}_N$, to obtain an N -dimensional vector signal.

It should be emphasized that the signal vector $\mathbf{s}(t)$ impinging on the elements of the ESPAR antenna is not measurable. This differs from the conventional adaptive array antennas where the received signal vector on the elements is observed. For the ESPAR antenna, only the single-port output $y(t)$ can be measured. More unfortunately, the single-port output $y(t)$ is a highly nonlinear function of \mathbf{x} , and includes an intractable matrix inverse, which makes it difficult to produce an analytical expression of adaptive performance. It is clear from Eq. (2.3) that each component of \mathbf{i} is not independent but mutually coupled with each other. The discussion above implies that a direct application of most of the algorithms of the conventional adaptive array to the ESPAR antenna is impractical.

Since the ESPAR antenna has much higher gain than omnidirectional antennas, the RF power necessary to transmit in wireless systems can be effectively reduced. Moreover, the ESPAR antenna consumes much lower power as well as, and can be fabricated much more cheaply.

Chapter 3

Likelihood Function

In this chapter, we will explain the purpose of the likelihood function for the SAGE algorithm.

The maximum likelihood [4, 5, 6] method is based on the idea that different populations generate different samples and that any given sample is more likely to have come from some populations than from others. To determine the estimation of the parameters signals, it would be necessary to determine a formula for the likelihood function and then maximize that function. Because likelihood is proportional to probability, we need to know the entire joint probability density function measurement in order to determine a formula for the likelihood function.

Let us consider a vector of unknown parameters Θ that describes a collection of N independent identically distributed $z(k)$ ($k = 1, 2, \dots, N$) and the vector $\mathbf{z}=(z(1), z(2), \dots, z(N))^T$. The likelihood of Θ , given the observations \mathbf{z} , is defined to be proportional to the value of the probability density function of the observations given the parameters, we have

$$l(\Theta|\mathbf{z}) \propto p(\mathbf{z}|\Theta) \propto p(z(1)|\Theta)p(z(2)|\Theta)\dots p(z(N)|\Theta),$$

because $z(i)$ are independent and identically distributed. Where l is the likelihood and p is the conditional joint probability density function. We could also write the likelihood with the natural logarithm when $p(z|\Theta)$ is an exponential function, in this case, we have

$$L(\Theta|\mathbf{z}) = \ln l(\Theta|\mathbf{z})$$

The important of maximum likelihood (ML) estimation is that it produces estimates that have very desirable properties. The ML estimates are consistent, asymptotically Gaussian and asymptotically efficient.

Chapter 4

SAGE Algorithm

4.1 Introduction to the SAGE Algorithm

Maximum likelihood (ML) estimation is the preferred signal parameter estimation technique. But a direct maximization of the likelihood function is often intractable due to its complexity. The expectation maximization (EM) method can facilitate maximizing likelihood functions that arise in statistical estimation problems. The EM algorithm iteratively alternates between an E-step, calculating the conditional expectation of the complete-data log-likelihood, and an M-step, simultaneously maximizing the expectation with respect to all the unknown parameters. But the EM algorithm has two drawbacks, i.e, the slow convergence due to the simultaneous updating of all parameters, and the high complexity to maximize the likelihood function. Therefore, the space-alternating generalized expectation-maximization (SAGE) algorithm [7] was introduced. Unlike subspace-based techniques like MUSIC and ESPRIT [8], the SAGE algorithm adopts a ML approach, i.e., it aims at maximizing some likelihood function.

4.2 Maximum Likelihood DoA and Delay Estimation

The SAGE algorithm uses a ML approach to estimate the parameters of the arriving signals. This set of parameters is denoted as

$$\Theta = (\alpha_1, \theta_1, \tau_1, \dots, \alpha_D, \theta_D, \tau_D).$$

The associated log-likelihood function is given by [5]

$$\Lambda(\Theta; \mathbf{y}(t)) = \sum_{n=1}^N 2\mathcal{R} \left\{ \left\langle y_n(t), \sum_{d=1}^D \mathbf{i}_n^T \mathbf{a}(\theta_d) \alpha_d x_d(t - \tau_d) \right\rangle \right\} - \sum_{n=1}^N \left\| \sum_{d=1}^D \mathbf{i}_n^T \mathbf{a}(\theta_d) \alpha_d x_d(t - \tau_d) \right\|_2^2,$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product in the L^2 -space, $\|f(t)\|_2 = \langle f, f \rangle^{\frac{1}{2}}$, and $\mathcal{R}\{\cdot\}$ is the real part operator. The ML estimate is given as

$$\hat{\Theta}_{\text{ML}} = \arg \max_{\Theta} \Lambda(\Theta; \mathbf{y}(t)). \quad (4.1)$$

4.3 The SAGE Algorithm

The SAGE algorithm evades the drawbacks of the EM algorithm, i.e., the low convergence rate and the high complexity to maximize the likelihood function with respect to the set of parameters [9]. The main purpose of the SAGE algorithm is to generalize the idea of data augmentation to simplify the computations of the EM algorithm.

To solve the equation (4.1), the SAGE algorithm breaks up the parameter estimation problem into several smaller problems by conditioning sequentially on a subset of the parameters and then using the EM to simplify the computation of each reduced problem. By introducing these parameter subsets, each of which constitutes a *hidden-data* space, we replace the maximization of the log-likelihood function for all the parameters by the maximization of a number of log-likelihood functions with reduced numbers of parameters.

4.3.1 Continues time

In every iteration, the SAGE updates a subvector $(\alpha_g, \theta_g, \tau_g)$ in Θ for a signal $x_g(t)$. After μ -th iteration, we compute

$$(\hat{\alpha}_g^{(\mu+1)}, \hat{\theta}_g^{(\mu+1)}, \hat{\tau}_g^{(\mu+1)}) = \arg \max_{(\alpha_g, \theta_g, \tau_g)} \left[\Lambda((\hat{\alpha}_1^{(\mu)}, \hat{\theta}_1^{(\mu)}, \hat{\tau}_1^{(\mu)}, \dots, \alpha_g, \theta_g, \tau_g, \dots, \hat{\alpha}_D^{(\mu)}, \hat{\theta}_D^{(\mu)}, \hat{\tau}_D^{(\mu)}); \mathbf{y}(t)) \right].$$

Omitting the terms which are independent of $(\alpha_g, \theta_g, \tau_g)$ we find

$$\begin{aligned}
(\hat{\alpha}_g^{(\mu+1)}, \hat{\theta}_g^{(\mu+1)}, \hat{\tau}_g^{(\mu+1)}) = & \\
& \arg \max_{(\alpha_g, \theta_g, \tau_g)} \left[\sum_{n=1}^N 2\mathcal{R} \{ \langle y_n(t), \mathbf{i}_n^T \mathbf{a}(\theta_g) \alpha_g x_g(t - \tau_g) \rangle \} \right. \\
& - \sum_{n=1}^N 2\mathcal{R} \left\{ \left\langle \sum_{\substack{d=1 \\ d \neq g}}^D \mathbf{i}_n^T \mathbf{a}(\hat{\theta}_d^{(\mu)}) \hat{\alpha}_d^{(\mu)} x_d(t - \hat{\tau}_d^{(\mu)}), \mathbf{i}_n^T \mathbf{a}(\theta_g) \alpha_g x_g(t - \tau_g) \right\rangle \right\} \\
& \left. - \sum_{n=1}^N \left\| \mathbf{i}_n^T \mathbf{a}(\theta_g) \alpha_g x_g(t - \tau_g) \right\|_2^2 \right]. \tag{4.2}
\end{aligned}$$

By introducing

$$y'_n(t) = y_n(t) - \sum_{\substack{d=1 \\ d \neq g}}^D \mathbf{i}_n^T \mathbf{a}(\hat{\theta}_d^{(\mu)}) \hat{\alpha}_d^{(\mu)} x_d(t - \hat{\tau}_d^{(\mu)}), \tag{4.3}$$

where $\hat{\alpha}_d^{(\mu)}, \hat{\theta}_d^{(\mu)}, \hat{\tau}_d^{(\mu)}$ are the estimated $\alpha_d, \theta_d, \tau_d$ after the μ -th iteration, we can transform the equation (4.2) into

$$\begin{aligned}
(\hat{\alpha}_g^{(\mu+1)}, \hat{\theta}_g^{(\mu+1)}, \hat{\tau}_g^{(\mu+1)}) = & \\
& \arg \max_{(\alpha_g, \theta_g, \tau_g)} \left[\sum_{n=1}^N 2\mathcal{R} \{ \langle y'_n(t), \mathbf{i}_n^T \mathbf{a}(\theta_g) \alpha_g x_g(t - \tau_g) \rangle \} \right. \\
& \left. - \sum_{n=1}^N \left\| \mathbf{i}_n^T \mathbf{a}(\theta_g) \alpha_g x_g(t - \tau_g) \right\|_2^2 \right]. \tag{4.4}
\end{aligned}$$

The equation (4.3) is referred to as the expectation (E) step and the equation (4.4) is the maximization (M) step. The flowchart of the SAGE algorithm is shown in Fig. 4.1. Note that K iteration cycles are employed to further improve the estimates.

4.3.2 Discrete time

As Matlab use only vector representation, we write the equations of the SAGE algorithm in the discrete time. As well as, we can apply directly the mathematical equations to the Matlab script.

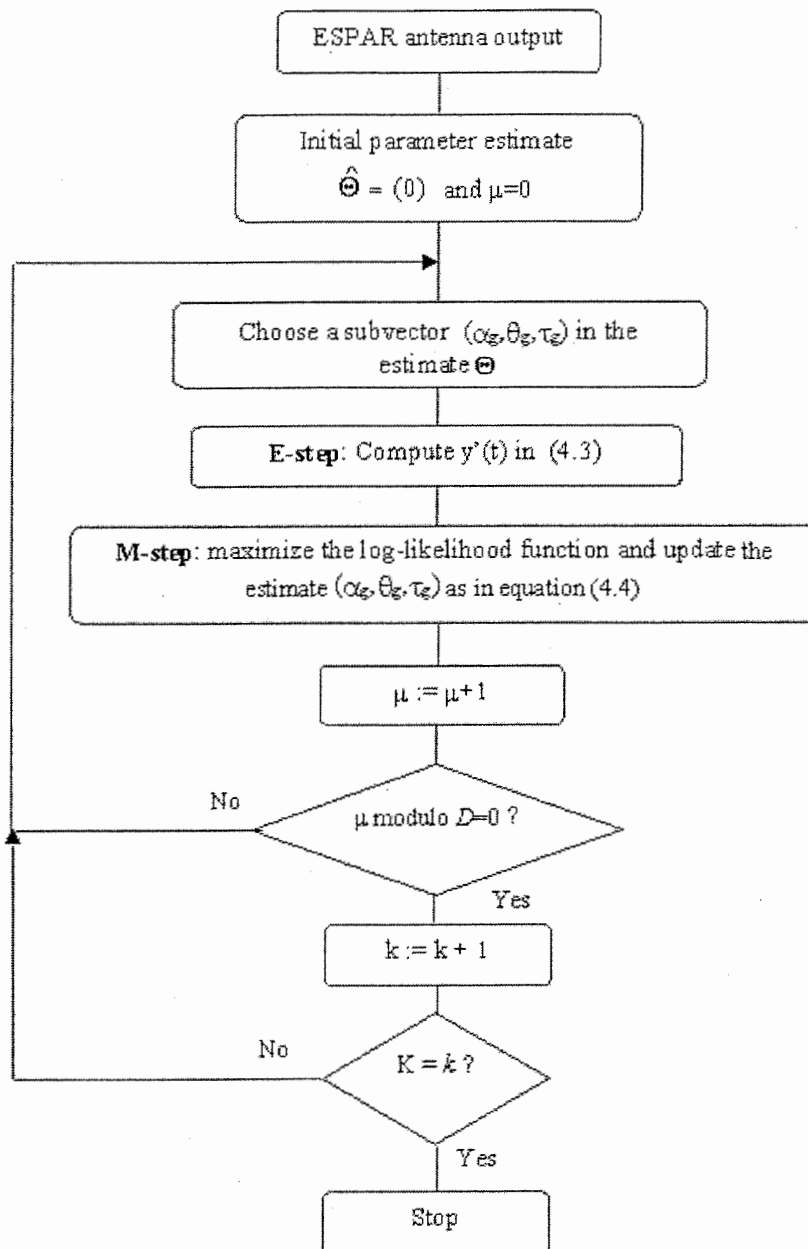


Figure 4.1: Flowchart of the SAGE algorithm for D arriving signals and K number of iteration cycles.

The ESPAR output is defined by:

$$\mathbf{Y} = \begin{bmatrix} y_{1,1} & \cdots & y_{1,N} \\ \vdots & \ddots & \vdots \\ \vdots & y_{k,n} & \vdots \\ \vdots & \cdots & \vdots \\ y_{K,1} & \cdots & y_{K,N} \end{bmatrix},$$

with

$$y_{k,n} = \sum_{d=1}^D \mathbf{C}(n, \theta_d) \alpha_d \mathbf{X}(d, (\tau_d + k) \text{ modulo } K) + v_{k,n},$$

where K is the number of sample of the antenna output, \mathbf{Y} a $K \times N$ matrix, \mathbf{C} is the equivalent steering $D \times N$ matrix of the receiving signal in respect of the patterns of the antenna. The $D \times K$ matrix \mathbf{X} denote the PN-sequence for all the signals. The script $\mathbf{X}(d, (\tau_d + k) \text{ modulo } K)$ denote a scalar from the PN-sequence of the d -th signal with a delay of τ_d . In the same way $\mathbf{C}(n, \theta_d)$ is the equivalent steering scalar for the angle θ_d and the n -th pattern. And we expressed the discrete log-likelihood function:

$$\Lambda = - \sum_{k=1}^K \sum_{n=1}^N \left| y_{k,n} - \sum_{d=1}^D \mathbf{C}(n, \theta_d) \alpha_d \mathbf{X}(d, (\tau_d + k) \text{ modulo } K) \right|^2.$$

Like in the continues time, we introduce the *hidden-data* space to simplify the computation of the log-likelihood function. Then, we note the log-likelihood function as follow,

$$\Lambda = - \sum_{k=1}^K \sum_{n=1}^N |y'_{k,n} - \mathbf{C}(n, \theta_g) \alpha_g \mathbf{X}(g, (\tau_g + k) \text{ modulo } K)|^2, \quad (4.5)$$

where

$$y'_{k,n} = y_{k,n} - \sum_{\substack{d=1 \\ d \neq g}}^D \mathbf{C}(n, \theta_d) \alpha_d \mathbf{X}(d, (\tau_d + k) \text{ modulo } K),$$

and the subscript g denotes the g -th signal which we want estimate its parameters. For a sake of simplicity, we derive the Eq. (4.5) in respect of the parameter α_g and we find an analytic expression for $\hat{\alpha}_g$:

$$\hat{\alpha}_g = \frac{1}{\sum_{n=1}^N |\mathbf{C}(n, \theta_g)|^2} \sum_{n=1}^N \mathbf{C}(n, \theta_g) \sum_{k=1}^K y'_{k,n}{}^* \mathbf{X}(g, (\tau_g + k) \text{ modulo } K),$$

The subscript * is the complex conjugate. But the problem is that we cant do same operation to find the DoA and the delay due to the high complexity of the log-likelihood function. Therefore, we estimate the delay and DoA by recursive try as shown in the flowchart Fig. 4.1. This mean that we search in the range of the delay and the angle the maximum of the log-likelihood function. So we could deduce the estimate value of the delay and angle.

Chapter 5

Computer Simulations

5.1 Improvement

The main problem of the SAGE algorithm is the time of computing due to the number of loop and the large size of the vectors. Therefore, we find a time improvement for the simulation. By analysis, the curves of the log-likelihood function, it's clear that when one estimate parameter is a good one, the numerical values of the log-likelihood function are much bigger than all parameters are wrong, as shown in the figure below.

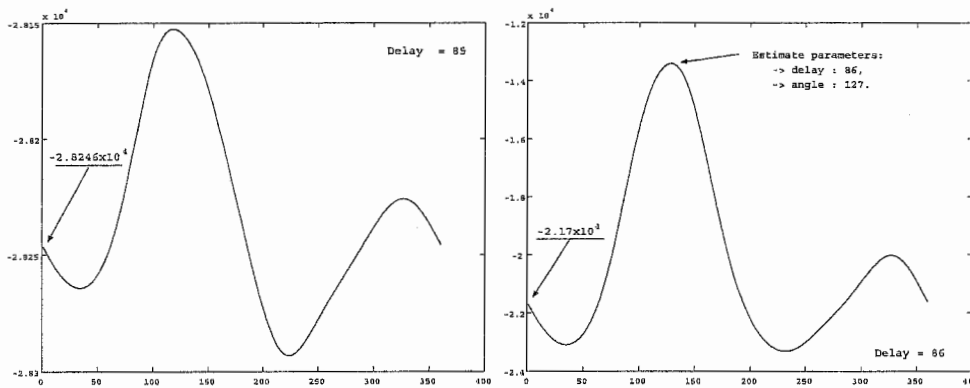


Figure 5.1: On the left, log-likelihood function where the delay are wrong and on the right, the delay is the good one. The value of the right parameters are : delay=86 and angle=127.

For a sake of simplicity, we represent the log-likelihood function by a $K \times 360$ matrix, where K is size of the PN-sequence and 360 denote the angles. Then the improvement consist to compare the actual value of the first column of the log-likelihood matrix, i.e., the angle equal

to one and for a certain delay, to the previous calculations always in the first column of the matrix. If the the value of the log-likelihood function for the angle one, is much higher than the others, we calculate the log-likelihood function for all the angle but if it is smaller, we stop the computing and we pass to the next delay. So, the time of computing is divide by three.

5.2 Sinusoidal Signals

At first, we employ the SAGE algorithm to estimate the DoAs and amplitudes of a number of simultaneously arriving sinusoidal signals on the basis of the ESPAR antenna output. The first simulation is conducted in order to determine the performance of the SAGE algorithm in a very simple case. We consider a scenario with one ESPAR antenna receiving only two signals with different DoAs (see Fig. 5.2). The polar plot provides the amplitude of every signal as the distance from the origin besides of the angle θ_d . For this simulation, the number

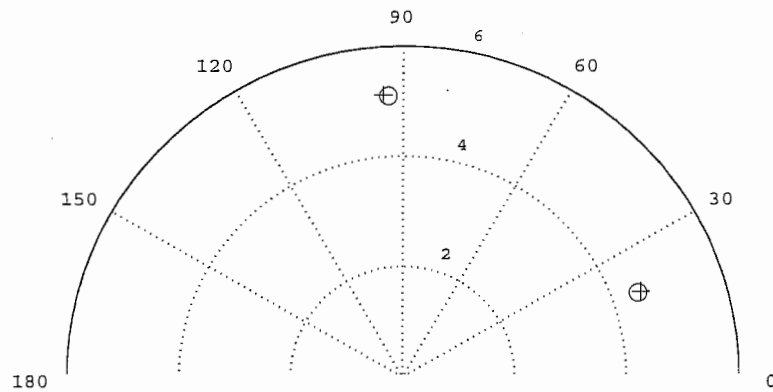


Figure 5.2: DoA and amplitude estimation for 2 sinusoid signals.
(+ : estimates, o : true values)

of patterns is $N=7$ and the number of the iteration cycles is $K=10$. The patterns of the ESPAR antenna determined by $\mathbf{i}_1, \dots, \mathbf{i}_N$ are randomly generated for the simulation. The true values in this simulation are given by Tab. 5.1.

Table 5.1: Amplitudes and DoAs.

Signal	Amplitude		Angle	
	True value	Estimation	True value	Estimation
1	$4-2i$	$4-2i$	20	20
2	$-1+5i$	$-0.9+5i$	93	94

As we can see, the estimates found by the SAGE algorithm are close to the true parameters. In the next simulation, we investigate the behavior of the algorithm for three arriving signals in the same conditions as the previous simulation, i.e., $N = 7$ and $K = 10$.

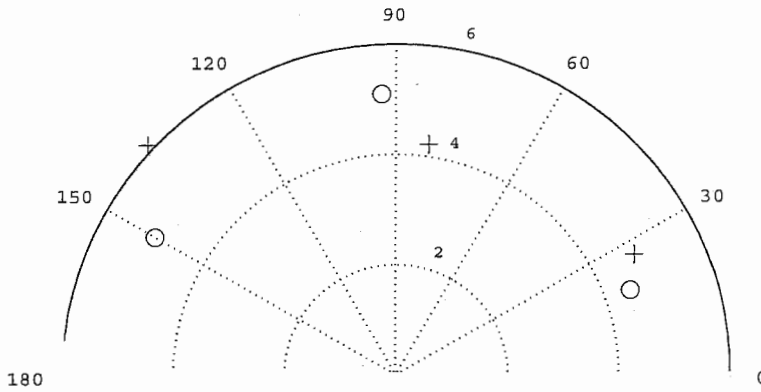


Figure 5.3: DoA and amplitude estimation for 3 sinusoid signals.
(+ : estimates, o : true values)

Table 5.2: Amplitudes and DoAs.

Signal	Amplitude		Angle	
	True value	Estimation	True value	Estimation
1	$4-2i$	$4.4-1.7i$	20	27
2	$-2+5i$	$-2+3.6i$	93	82
3	$3+4i$	$-3.2+5.1i$	150	137

Obviously, in Fig. 5.3, the performance of the SAGE algorithm is

poor for the case of 3 impinging signals. This is due to the high correlation between the arriving signals.

5.3 CDMA Signals

In code division multiple access (CDMA) systems, the information is modulated onto pseudo-noise (PN) sequences, which are known by the receiver. We consider such a scenario, letting the PN-sequences be randomly generated. For the sake of simplicity, the size of the PN-sequences is fixed to 100. We make the assumption that the receiver knows the PN-sequences. Now, the original d -th signal $x_d(t)$ reads

$$x_d(t) = \sum_{k=-\infty}^{+\infty} \sum_{l=0}^{p-1} q_l^{(d)} r(t - lT_c - kpT_c),$$

with $p=100$ defining the length of the PN-sequence. The vector $(q_0^{(d)}, \dots, q_{(p-1)}^{(d)})$ is the PN-sequence of the d -th signal where $q_l^{(d)} \in [-1, 1]$. Furthermore, the chip pulse of every signal $x_d(t)$ is defined as

$$r(t) = \begin{cases} 1, & t \in [0, T_c) \\ 0, & \text{otherwise} \end{cases},$$

where T_c is the chip length. Here, we choose $K=5$. We also use only seven patterns, i.e., $N=7$. For one and two arriving signals, the SAGE algorithm again turns out to provide good results. In the next simulation, we make the assumption that the number of arriving signals is equal to four. As we can see, in the Fig. 5.4 and Tab. 5.3, the estimated values are very close to the true values.

In the next simulation, we investigate the behavior of the algorithm when the number of arriving signals is high, i.e., sixteen arriving signals. The results of the simulations are presented in Fig. 5.5 and Tab. 5.4. The last simulation shows us that the algorithm works very well with a large number of signals. In fact, with PN-sequences, the algorithm can distinguish the signals much easier than with sinusoidal signals.

For all simulations, the polar plot provides the delay $\tau_d \in [0, T_S)$ of every signal as the distance from the origin besides of the angle θ_d .

We can see in Tab. 5.4 that all the parameter estimates are close to the true values. We conclude that the SAGE algorithm applied for the ESPAR antenna works even on a large number of arriving signals. However, the convergence of the SAGE algorithm is not guaranteed, as can be seen in Tab. 5.4 for the DoA of the first signal. But this does

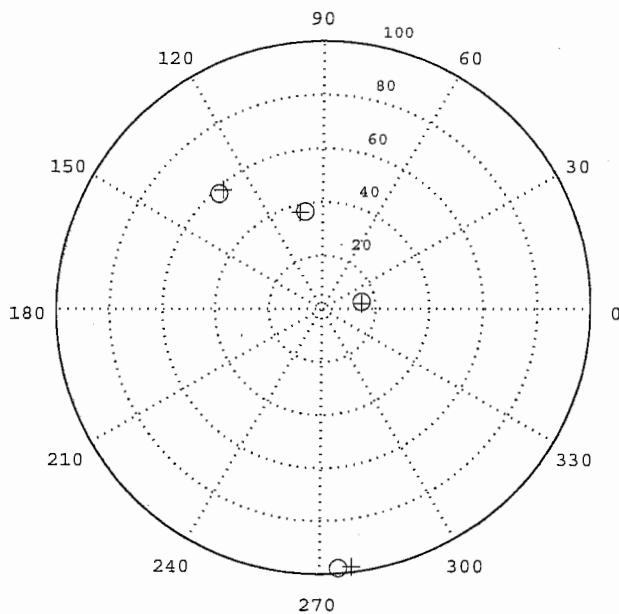


Figure 5.4: DoA and delay estimation for 4 CDMA signals.
 (+ : estimates, o : true values)

Table 5.3: Values of the amplitudes, angles and delays.

Signal	Amplitude		Angle		Delay	
	T.V.	E.	T.V.	E.	T. V.	E
1	$3-2i$	$2.8-2.3i$	10	7	15	15
2	$-7+1i$	$-7.3+1i$	100	103	37	37
3	$-2+6i$	$-2+6.2i$	132	130	58	58
4	$-2-5i$	$-2+5i$	274	270	98	98

not affect the estimation of the others parameters.

In the next simulation, we investigate the influence of the noise on the SAGE algorithm. Therefore, we choose for a signal to noise ratio (SNR) of 25, 8 and 0 dB. In this simulation, we only focus on the estimated value of the DoA. The SNR is defined as the ratio of the energy of the antenna output signal over the variance of the noise.

Tab. 5.5 shows the results for these different SNRs. Obviously,

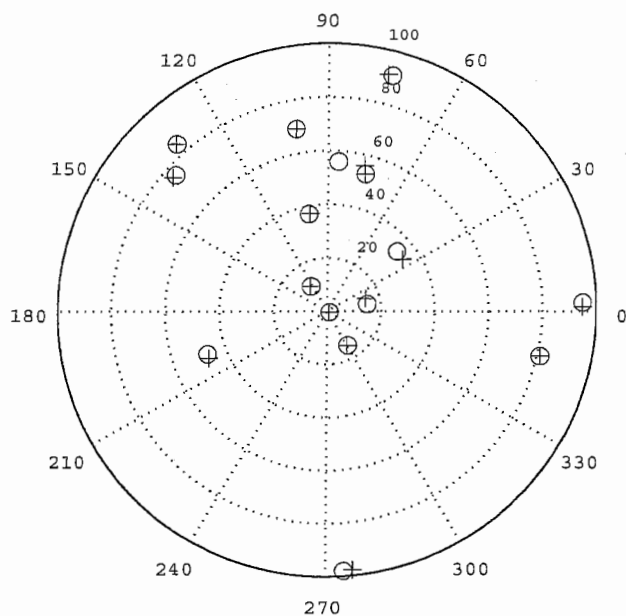


Figure 5.5: DoA and delay estimation for 16 CDMA signals.
 (+ : estimates, o : true values)

Table 5.4: Values of the amplitudes, angles and delays.

Signal	Amplitude		Angle		Delay	
	T.V.	E.	T.V.	E.	T. V.	E
1	3-2i	3.2-1.8i	10	15	15	15
2	-7+1i	-7+1.2i	100	100	69	69
3	-2+6i	-2.1+5.8i	132	132	84	84
4	-2-5i	-1.7-5i	274	276	98	98
5	10-1i	9.8-1.5i	301	300	15	15
...	...					
16	6+10i	6+9.8i	123	123	11	11

when the SNR increases, the precision of the estimation values becomes better.

In the next simulation, the arriving signals are very close to each others. The polar plot in Fig. 5.6 shows the results of the simulation. As we can see in the Fig. 5.6, the SAGE algorithm finds the delay accurately, whereas the DoA estimates exhibit some deviations.

Table 5.5: DoA and delay estimation for different SNR.

	DoA	Estimated DoA.		
		SNR=25 dB	SNR=8 dB	SNR=0 dB
1	114	114	110	126
2	25	25	27	31
3	315	316	313	307
4	246	245	244	249

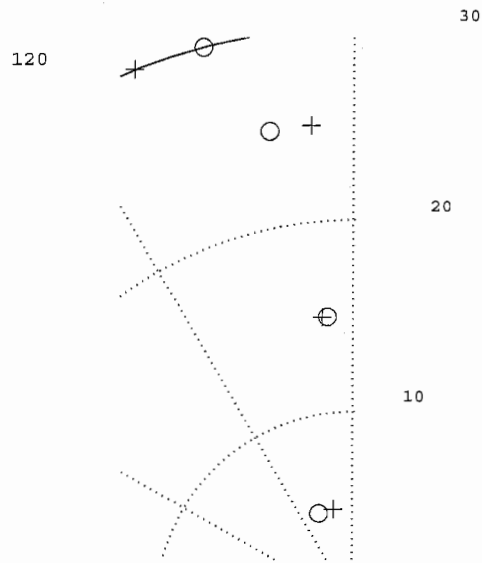


Figure 5.6: DoA and delay estimation of nearby arriving signals.
(+ : estimates, o : true values)

To more precisely assess the DoA and delay estimation errors for different PN-sequence lengths and different SNRs we define the estimation error as

$$E = \frac{1}{D} \sum_{d=1}^D |s_d - \hat{s}_d|,$$

where s_d is true value of the DoA or the delay for the d -th signal and \hat{s}_d is the estimation of the parameter. In this simulation, the number of arriving signals is $D = 6$. The range of the SNR is from 3 dB to 57

dB and the length of the PN-sequence is between 16 and 100.

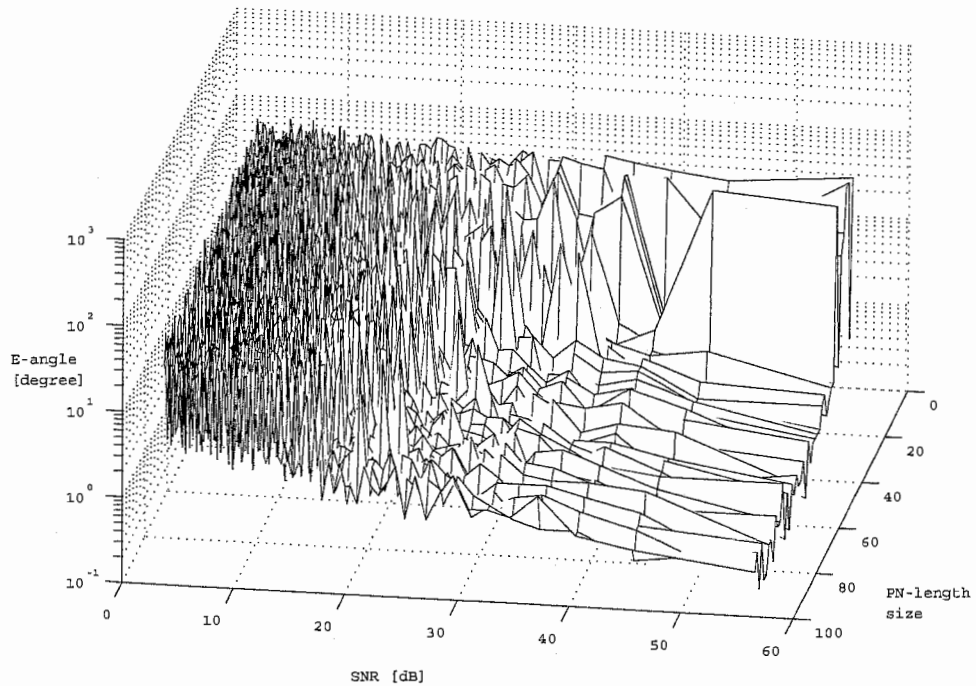


Figure 5.7: DoA estimation errors.

Obviously in Fig. 5.7 and in Fig. 5.8, the error E grows as both the SNR and the PN-sequence length decreases. The worst case is to have a low SNR and a short PN-sequence.

5.4 Conclusion

We have proposed in this chapter the SAGE algorithm for the ESPAR antenna. The simulations show that our proposed algorithm can be used with an ESPAR antenna to estimate the DoAs and delay of impinging signals.

We have shown in the first simulation that the estimation of sinusoid signals is difficult due to the high correlation between the arriving signals. For one or two arriving signals SAGE algorithm can

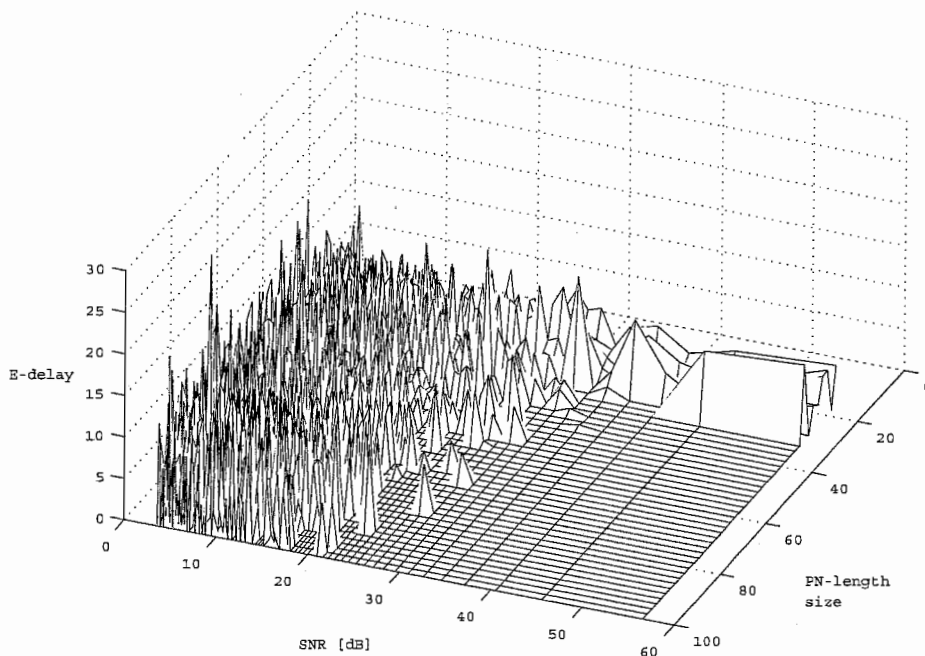


Figure 5.8: Delay estimation errors.

estimate the parameters with a good accuracy. But the resolution of the algorithm is poor.

By introducing the PN-sequences we can access to better results especially for a large number of signals. A simulation with twenty arriving signals and reasonable SNR, show us that the SAGE algorithm give a very high accuracy estimation of the signals parameters. Therefore, precision and resolution are good. The precision of the SAGE algorithm is about one degree and the resolution is less than three degrees. However, we have noted that the delay estimation give better results than DoA estimation. Furthermore, the convergence of the log-likelihood function is not guaranteed for each arriving signal but this does not affect the estimation of the others parameters signals.

Obviously, the estimation results become better as the SNR increases but it depends also of the length of the PN-sequence. This mean if the PN-sequence is too short even for a high SNR the estimation of the parameters is not good. In the case of six arriving signals,

a reasonable SNR and a twenty PN-sequence length, we obtain a good estimation of the parameters.

Conclusion

In this paper, we have applied the SAGE algorithm to estimate the DoAs, amplitudes and delays of multiple arriving signals on an ESPAR antenna. Computer simulations have shown that this set-up enables an accurate estimation of the DoAs and amplitudes of two arriving sinusoidal signals. Furthermore, in the case of PN-sequences of e.g. length one hundred and negligible noise, twenty signals can be reliably resolved, including their time delays. The precision of the estimates becomes better as the SNR increases. Due to the use of an ESPAR antenna, the proposed technique is for instance a cost-efficient alternative for locating mobile terminals in cellular systems.

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Annexe 1 : MatLab source codes

File name: patterns.m,

```

%*****
%*
%* Main goal:
%*
%* Create randomly some weight vector in respect of the
%* architecture of the antenna.
%*
%*****

function [W] = patterns(nrPatterns)

% 7-element ESPAR antenna configuration
ESPARx0 = 50; ESPARy00 = 0.0008616-i*0.0120795; ESPARy10 =
-0.0006963+i*0.0036462; ESPARy11 = 0.0044216-i*0.0071600;
ESPARy21 = 0.0009721+i*0.0047851; ESPARy31 =
-0.0005376-i*0.0011297; ESPARy41 = 0.0001701-i*0.0002950; ESPARy
= [ESPARy00 ESPARy10 ESPARy10 ESPARy10 ESPARy10 ESPARy10 ESPARy10;
    ESPARy10 ESPARy11 ESPARy21 ESPARy31 ESPARy41 ESPARy31 ESPARy21; ...
    ESPARy10 ESPARy21 ESPARy11 ESPARy21 ESPARy31 ESPARy41 ESPARy31; ...
    ESPARy10 ESPARy31 ESPARy21 ESPARy11 ESPARy21 ESPARy31 ESPARy41; ...
    ESPARy10 ESPARy41 ESPARy31 ESPARy21 ESPARy11 ESPARy21 ESPARy31; ...
    ESPARy10 ESPARy31 ESPARy41 ESPARy31 ESPARy21 ESPARy11 ESPARy21; ...
    ESPARy10 ESPARy21 ESPARy31 ESPARy41 ESPARy31 ESPARy21 ESPARy11];
ESPARZ = inv(ESPARy); ESPARminx = -300; ESPARmaxx = 300;

ESPARX = (ESPARmaxx-ESPARminx)*rand(nrPatterns, size(ESPARZ,
1)-1)+ESPARminx;

W = zeros(size(ESPARX, 2)+1, nrPatterns); for ii = 1 : nrPatterns
    aw = inv(ESPARZ+diag([ESPARx0 i*ESPARX(ii, :)]))*...
        [1; zeros(size(ESPARX, 2), 1)];
    W(:, ii) = aw/sqrt(aw'*aw);
end

```


Annexe 2 : MatLab source codes

File name: gen_u_theta.m,

```
%*****  
%* *  
%* Main goal: *  
%* *  
%* Computation of the output vector in respect to the angle. *  
%* *  
%*****  
  
function [u_theta] = gen_u_theta(val_angle,W)  
  
ESPARpos = [0 exp(i*pi*(0:1/3:1.9))/4];  
  
u_theta = exp(-i*2*pi*((ones(size(val_angle,2),  
1)*abs(ESPARpos)).*cos(ones(size(val_angle,2),  
1)*angle(ESPARpos)-...  
(val_angle'*pi/180)*ones(1, length(ESPARpos))))))*W;
```


Annexe 3 : MatLab source codes

File name: SAGE.m,

```
*****
%* Mathieu LENOBLE - ATR - Internship *
%* * *
%* SAGE Algorithm using "sinusoidal signals" *
%* * *
%* ENSSAT - 2003 *
%* * *
*****

%* * *
%* Main goal: *
%* * *
%* SAGE algorithm adopts a maximum likelihood approach. *
%* It aims at maximizing some likelihood function. *
%* * *
%* * *
*****

%* * *
% Mathlab variables :
%
clc;
close all;
clear all;

%* * *
% Program constant :
%
nrPatterns=7; % Number of patterns
nrOfSample=50; % Number of samples

%* * *
% Signals parameters :
%
% Amplitudes and angles of the arriving signals,
```

```

parameters=[4-2*i,10,-1+5*i,145,3+4*i,75,1-5*i,220,1-3*i,290,-5-2*i,300];
parameters2=parameters;
si=size(parameters,2);

%*****
%                               Antenna parameters :

W = patterns(nrPatterns);           % Weight vector
u_theta_ESPAR=gen_u_theta(parameters(2:2:si),W); % Output vector in
                                           % respect to the angle

%*****
%                               Noise parameters :

SNR = 1;           % Signal to noise ratio (not in dB!)

%*****
%                               Antenna output signal :

parameters=parameters2;
for jj=1:nrOfSample
    noise=sqrt(SNR*0.5)*(randn(1,nrPatterns)+i*randn(1,nrPatterns));
    Y(jj,:)=parameters(1:2:si)*u_theta_ESPAR+noise;
end

%*****
%                               SAGE algorithm :

% Initialisation,
parameters=zeros(1,si);

for nrLoop=1:10
    for p=1:2:si

        % E-step :
        osignal=0;
        for ii=1:2:si
            if ii~=p
                osignal=osignal+parameters(ii)*gen_u_theta(parameters(ii+1),W);
            end
        end
        end

        % M-step :
        for angle=1:360
            u_theta=gen_u_theta(angle,W);
            alpha=(sum(Y)-nrOfSample*osignal)*u_theta'/(nrOfSample*u_theta*u_theta');
            L=0;
            for k=1:nrOfSample
                L=L-(Y(k,:)-alpha*u_theta-osignal)*(Y(k,:)-alpha*u_theta-osignal)';
            end
        end
    end
end

```

```

        end
        if angle==1
            L_max=L;
        elseif L_max<L
            L_max=L;
            parameters(p:p+1)=[alpha,angle];
        end
    end
end
end
end

%*****
%           Display of the estimation and the right parameters :

% Good parameters in red cross,
polar(parameters2(2:2:si)*pi/180,abs(parameters2(1:2:si)),'r+')
hold on
% Estimation parameters in blue,
polar(parameters(2:2:si)*pi/180,abs(parameters(1:2:si)),'+')

```


Annexe 4 : MatLab source codes

File name: SAGE_cplx.m,

```
%*****  
%* Mathieu LENOBLE - ATR - Internship *  
%* *  
%* SAGE Algorithm using "CDMA signals" *  
%* *  
%* ENSSAT - 2003 *  
%*****  
  
%*****  
%* *  
%* Main goal: *  
%* *  
%* SAGE algorithm adopts a maximum likelihood approach. *  
%* It aims at maximizing some likelihood function. *  
%* *  
%*****  
  
%*****  
% Mathlab variables :  
  
close all;  
clear all;  
clc;  
  
%*****  
% Program constant :  
  
nrPatterns=7; % Number of patterns  
nrSample=100; % Number of samples  
  
%*****  
% Signals parameters :  
  
% Amplitudes of the arriving signals,
```

```

amplitude2=[3-2*i,-7+1*i,-2+6*i,-2-5*i,5+9*i,1-7*i];
nrSignal=size(amplitude2,2);

% Angles randomly generated,
angle2=round(1+(360-1)*rand([1,nrSignal]));

% Delays randomly generated,
delay2=round(1+(nrSample-1)*rand([1,nrSignal]));

%*****
%                               Antenna parameters :

W=patterns(nrPatterns);                % Weight vector
u_theta_ESPAR = gen_u_theta(angle2,W);  % Output vector in respect to the angle
C=gen_u_theta([1:360],W);
X=sign(rand(nrSignal,nrSample)-0.5);    % PN-sequence
X=cat(2,X,X);

%*****
%                               Noise parameters :

SNR = 20;                               % Signal to noise ratio (not in dB!)

%*****
%                               Antenna output signal :

noise=sqrt(SNR*0.5)*(randn(nrSample,nrPatterns)+i*randn(nrSample,nrPatterns));
Y=0; for ii=1:nrSignal
    Y=Y+transpose(X(ii,delay2(ii):delay2(ii)+nrSample-1))*amplitude2(ii)*...
        u_theta_ESPAR(ii,:);
end Y=Y+noise;

%*****
%                               SAGE algorithm :

% Initialisation,
amplitude=zeros(1,nrSignal);
angle=zeros(1,nrSignal);
delay=ones(1,nrSignal);

for nrLoop=1:5
    for pp=1:nrSignal

        % E-step :
        Yinter=0;
        Cin=gen_u_theta(angle,W);
        for ii=1:nrSignal
            if ii~=pp
                Yinter=Yinter+transpose(X(ii,delay(ii):delay(ii)+...

```

```

        nrSample-1))*amplitude(ii)*Cin(ii,:);
    end
end
Y1s=Y-Yinter;

% M-step :
for phi=1:nrSample
    rho=X(pp,phi:phi+nrSample-1)*conj(Y1s);
    for theta=1:360
        % Calculation of the alpha estimate,
        alpha(phi,theta)=conj(rho*transpose(C(theta,:)))/(nrSample*...
            sum(abs(C(theta,:)).*abs(C(theta,:))));
        % Calculation of the Log-likelihood function
        Linter=Y1s-alpha(phi,theta)*transpose(X(pp,phi:phi+...
            nrSample-1))*C(theta,:);
        L(phi,theta)=-sum(sum(abs(Linter).*abs(Linter)));

        % Improvement of the algorithm :
        if theta==1
            if phi==1
                Lmax1=L(phi,theta);
            else
                if L(phi,theta)>Lmax1
                    Lmax1=L(phi,theta);
                else
                    L(phi,2:360)=-10^9;
                    break
                end
            end
        end
    end
    %*****
end
end

% Find the angle and delay optimum,
[phi_opt,theta_opt]=find(L==max(max(L)));

% Update of the parameters,
amplitude(pp)=alpha(phi_opt,theta_opt);
angle(pp)=theta_opt;
delay(pp)=phi_opt;

% New cycle
clear L Linter Lmax1;
end
end

%*****
%           Display of the estimation and the right parameters :

```

```
% Good parameters in red cross,  
polar(angle2*pi/180,delay2,'r+') hold on  
% Estimation parameters in blue,  
polar(angle*pi/180,delay,'+')
```