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Abstract

High-resolution algorithms for the detection and estimation of Directions Of Arrival (DOA) such as MUSIC, lead to accurate results but require the computation of the noise-subspace through an expensive covariance matrix eigendecomposition. Adaptive estimators of the noise-subspace can be very useful in a non-stationary environment when the convergence is possible with a few number of snapshots. Some adaptive methods are presented showing that an indirect noise-subspace estimation through a signal subspace estimation can be advantageous both in terms of convergence rate and computation complexity during each update. Some computer simulations examples showing performances are provided.

Table of contents

1. Introduction

High-resolution algorithms for the detection and estimation of Directions Of Arrival (DOA) lead to accurate results but require the computation of the noise-subspace through an expensive covariance matrix eigendecomposition. In applications using sensor array processing, the knowledge of the DOA's of the desired signals and interferences is important. Once DOA's are known with sufficient accuracy, there exist several efficient beamforming techniques [1] that permit to steer deep nulls towards interferences and narrow beams towards desired signals. Generally, this leads to an increase of the channel capacity with the possibility of frequency re-use for instance in cellular mobile communications. The most accurate DOA findings methods so far are the so-called eigenspace high-resolution algorithms such as MUSIC, ESPRIT, Weighted Subspace Fitting [8]-[10]. However, these techniques are based on off-line computation of the eigenvectors of an estimate of signal covariance matrix, making them unsuitable for adaptive processing needed in the tracking of varying parameters.

In [2], some adaptive algorithms leading to an estimate of the signal or noise-subspace were derived. Adaptive estimators of the noise-subspace (used in particular, for the MUSIC algorithm) can be very useful in a non-stationary environment when the convergence is possible with a few number of snapshots. Eventhough some theoretical convergence rates were developed for the proposed algorithms, no complexity analysis was shown in terms of the minimum number of snapshots or sensors needed for the estimate of the subspace to yield satisfactory detections. In fact, it was stated that the relations of these parameters were too complicated for a useful analysis especially concerning the number of snapshots. If this number is too large, a direct eigendecomposition would be preferable, and there would be no need of an adaptive procedure in the first place.

In this technical report, the adaptive signal eigenspace proposed in [2] is used to determine indirectly the noise-subspace of the MUSIC spectrum. In fact, it was found that a direct adaptive computation of the noise-subspace lead to poor convergence rates and even numerical problems as will be shown in the next section. The new approach is compared to the direct one and found more reliable especially for moderate and high values of the SNR. Furthermore, several computer simulations are provided for a specific sensor array system showing the performance of the adaptive MUSIC spectrum estimate for varying values of the number of sensors and SNR. In particular, the minimum number of snapshots resulting in adequate detection is derived for each sensor array communications scenario leading to the computation of the number of operations for the proposed aproaches and compared to that of conventional adaptive methods. In the simulation examples, it is assumed that the number of impinging signals is fixed and signals are narrow-banded and non-coherent. Finally, a simple efficient beamforming technique is presented completing the proposed adaptive sensor array beamforming system.

Notations

Uppercase and lower case bold letters denote respectively matrices and vectors, the symbol superscript $\dagger$ means vector or matrix transpose conjugate.

2. Adaptive eigensubspace derivation techniques

An array of m sensors receive d narrow-band plane waves from far-field emitters. The m-vector of sensor outputs is modeled by the following equation:

$$\mathbf{x}(t)=\mathbf{A}(\theta)\mathbf{s}(t)+\mathbf{n}(t) \tag{1}$$

The real d-vector θ corresponds to the unknown signal parameters and the columns of the m by d matrix $\mathbf{A}(\theta)$ are the array response vectors

$$\mathbf{A}(\theta)=\begin{pmatrix}\mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_d)\end{pmatrix} \tag{2}$$

The d-vector $\mathbf{s}(t)$ contains the complex envelopes of the emitter signals and $\mathbf{n}(t)$ is a complex m-vector of additive white noise. Let N be the number of collected snapshots such that:

$$\mathbf{X}=\begin{pmatrix}\mathbf{x}(t_1) & \cdots & \mathbf{x}(t_N)\end{pmatrix} \tag{3}$$

The eigendecomposition of the sample correlation matrix leads to:

$$\hat{\mathbf{R}}=\frac{1}{N}\mathbf{X}\mathbf{X}^\dagger=\sum_{i=1}^{m}\lambda_i\mathbf{v}_i\mathbf{v}_i^\dagger \tag{4}$$

where λ_i are the eigenvalues arranged in descending order and v_i are the corresponding eigenvectors. It is known that the smallest $m-d$ eigenvalues $\lambda_{d+1},\cdots,\lambda_m$ will be approximately equal, close to the noise power. Furthermore for non-coherent signals, the MUSIC spectrum defined by:

$$S(\theta)=\frac{\mathbf{a}^\dagger(\theta)\mathbf{a}(\theta)}{\mathbf{a}^\dagger(\theta)\mathbf{V}_n\mathbf{V}_n^\dagger\mathbf{a}(\theta)} \tag{5}$$

exhibits minimae at the correct DOA's where $\mathbf{V}_n=\begin{pmatrix}\mathbf{v}_{d+1} & \cdots & \mathbf{v}_m\end{pmatrix}$ ($\mathbf{V}_n\mathbf{V}_n^\dagger$ is called the noise-subspace). The high-resolution capabilities pertain to the possibility of distinguishing closely spaced DOA's.

Due to the considerable amount of computations needed to find the eigendecomposition (in this case, we need an order of m^3 operations), applications so-far have been limited to off-line processing. Some family of adaptive algorithms to estimate the noise-subspace have been proposed in [2]-[5]. For moderate to high SNR (> 15 dB values which are very common in real situations), a large number of snapshots are generally needed for the convergence to occur. It is probably due to the fact that the noise-subspace is formed with the eigenvectors corresponding to the smallest eigenvalues $\lambda_{d+1},\cdots,\lambda_m$ all closely equal to the noise power which in the case of moderate to high SNR will be negligible with respect to the signal eigenvalues. Consequently, the condition number (ratio of largest to smallest eigenvalue) of the covariance matrix would be large resulting in known numerical problems [6], [11] during the update process. For a sufficient convergence to take place, a significant increase in the number of updates will be needed therefore nullifying the original purpose of savings in the number of computations. On the other side, adaptive signal subspace techniques converging to the eigenvectors corresponding to the highest eigenvalues seem to result in accurate estimates in relatively much faster time. This is presumably due to the well-behaved numerical properties for the obtention of the highest eigenvalues. Moreover, in situations where $d<<m$, the number of signal eigenvectors (d) we have to compute is much less than the noise eigenvectors ($m-d$) so that an additional savings of computations result. The problem is how do we get from those eigenvectors the noise-subspace. The answer lies in the relationship between the ensemble of these eigenvectors. The collection of all eigenvectors $\mathbf{V}=(\mathbf{v}_1 \quad \cdots \quad \mathbf{v}_m)$ is in fact a unitary matrix due to the hermitian property of the

correlation matrix. This implies that: $\begin{pmatrix}\mathbf{V}_s & \mathbf{V}_n\end{pmatrix}\begin{pmatrix}\mathbf{V}_s^\dagger \\ \mathbf{V}_n^\dagger\end{pmatrix}=\mathbf{I}$ (where $\mathbf{I}$ is the identity matrix), thus $\mathbf{V}_n\mathbf{V}_n^\dagger=\mathbf{I}-\mathbf{V}_s\mathbf{V}_s^\dagger$ and the MUSIC spectrum of equation (5) becomes:

$$S(\theta)=\frac{\mathbf{a}^\dagger(\theta)\mathbf{a}(\theta)}{\mathbf{a}^\dagger(\theta)\mathbf{a}(\theta)-\mathbf{a}^\dagger(\theta)\mathbf{V}_s\mathbf{V}_s^\dagger\mathbf{a}(\theta)} \quad (6)$$

Therefore, the MUSIC spectrum in equation (5) could be derived as well from the signal subspace, and in fact, it could be computationally and numerically more advantageous to do so in an adaptive algorithm. The adaptive method to obtain the signal eigenvectors follow that of [2] basically as it resulted in good estimates using two versions of the gradient approach. It can be shown that the maximum of the cost function $J(\mathbf{V})=Trace\{\mathbf{V}^\dagger\mathbf{R}\mathbf{V}\}$ subject to the constraint $\mathbf{V}^\dagger\mathbf{V}=\mathbf{I}$ where $\mathbf{V}$ is m by d and $\mathbf{R}$ is the true correlation matrix, is obtained for $\mathbf{V}=\mathbf{V}_s$, the collection of eigenvectors corresponding to the highest eigenvalues. The optimization process can be accomplished via a constrained gradient search procedure such that $\mathbf{V}_s'(k)=\mathbf{V}_s(k-1)+\mu\nabla(k)$ with $\mathbf{V}_s(k)$ is obtained through the orthogonalization of the columns of $\mathbf{V}_s'(k)$ (by using standard procedures such as the Gram-Schmidt or the QR factorization). The gradient of $J(\mathbf{V}_s)$ is: $\nabla J=\frac{\partial}{\partial\mathbf{V}_s}\{Tr\{\mathbf{V}_s^\dagger\mathbf{R}\mathbf{V}_s\}\}=2\mathbf{R}\mathbf{V}_s$. In practice, we use the sample covariance matrix $\hat{\mathbf{R}}$ instead of $\mathbf{R}$. We update it in a non-stationary environment as follows:

$$\hat{\mathbf{R}}(k)=(1-\alpha)\hat{\mathbf{R}}(k-1)+\alpha\mathbf{x}(t_k)\mathbf{x}^\dagger(t_k) \quad (7)$$

where α is the forgetting factor satisfying $0<\alpha<1$.

2.1. First approach:

After replacing $\nabla_1 J(k) \approx 2\hat{\mathbf{R}}(k)\mathbf{V}_s(k-1)$ into the gradient equation, the signal eigenspace update becomes as follows $\mathbf{V}_s'(k) = (\mathbf{I} + 2\mu\hat{\mathbf{R}}(k))\mathbf{V}_s(k-1)$. In practice, we can still further simplify this update by using directly

$$\mathbf{V}_s'(k) = \hat{\mathbf{R}}(k)\mathbf{V}_s(k-1) \tag{8}$$

Result due to the fact that $\mathbf{I} + 2\mu\hat{\mathbf{R}}(k)$ and $\hat{\mathbf{R}}(k)$ have identical eigenvectors, then

$\mathbf{V}_s(k)$ =first d columns of **Q**-matrix of the QR factorization of $\mathbf{V}_s'(k)$ (9)

Note: In a non-stationary case, letting $\alpha = \frac{1}{k}$ for $k = 1,\ldots,N$ during the eigenspace estimation procedure converges exactly at time t_N to the sample correlation matrix shown in equation 4.

2.2. Second approach:

Letting $\alpha = 1$ results in an instantaneous LMS-type gradient estimator $\nabla_2 J(k) \approx 2\mathbf{x}(k)\mathbf{y}^\dagger(k)$ where $\mathbf{y}^\dagger(k) = \mathbf{x}^\dagger(k)\mathbf{V}_s(k-1)$. Following the choice of the step size of [2], we obtain the normalized LMS-type algorithm

$$\mathbf{V}_s'(k) = \mathbf{V}_s(k-1) + 2\mathbf{x}(k)\mathbf{y}^\dagger(k)\mathbf{M}(k) \tag{10}$$

where $\mathbf{M}(k)$ is a d by d diagonal matrix whose elements are $\mathbf{M}_i(k) = \mu\left|\frac{\mathbf{y}_d(k)}{\mathbf{y}_i(k)}\right|$ and $\mathbf{y}_i(k)$ is the i^{th} element of the d by 1 vector $\mathbf{y}(k)$. In the simulation examples, we used $\mu = \frac{2}{3m}$ suggested in [6] and the QR factorization to orthogonolize $\mathbf{V}_s'(k)$.

3. Computational complexity analysis

As in Reference [2], we counted the multiplication of two complex numbers as one operation and a multiplication of a real number by a complex number as one-half operation. We will not provide the approximate convergence rates since they are already developed in [2], rather we will emphasize the dependence of the resulting signal-subspace estimates with respect to the number of used snapshots.

<u>First approach:</u>

The required number of operations per adaptation cycle is as follows:

∗ for the updated covariance matrix in equation (7): $\frac{3m^2}{4}+\frac{m}{2}$

∗ for the adaptation equation (8): m^2d

∗ for the orthogonalization procedure in equation (9) either Gram-Schmidt or the QR factorization method could be used as both of them require md^2 operations. In our case, we preferred using the QR method since it tends to be numerically more robust and also, lends itself more easily to parallel implementations [7] which will make the throughput rate even higher.

<u>Second approach:</u>

In this case, the updated covariance step is not needed as the instantaneous data snapshots are used directly in the adaptation equation. Therefore, we need:

∗ $2md+2d$ operations in equation (10)

∗ similarly as before, md^2 operations for the QR transform.

For both approaches, the count of operations is for one adaptation cycle and for a correct comparison with a direct eigendecomposition in terms of computational complexity, one has to multiply the previous numbers by the minimum required number of adaptation cycles in order for the convergence towards the eigenspace estimate to take place. Taking into account the number of snapshots N, the total computation cost is for the first approach $\left(\frac{3m^2}{4}+\frac{m}{2}+m^2d+md^2\right)N$, and for large m, the dominant term is m^2dN. This can be compared to a minimum of $\frac{4}{3}m^3$ operations [11, Chapter 11] for a direct eigendecomposition. It is clear that as long as $dN<m$ then this approach might be useful. In the second approach, the operation count is $\left(2md+2d+md^2\right)N$ and the method is useful for $d^2N<m^2$.

An analytical relationship between the rate of convergence and the required parameters m, d and the SNR is often difficult to formulate. It is known that, in the case of the general convergence properties of the LMS algorithm [6], the larger will be the condition number (ratio between largest to smallest eigenvalue) of the correlation matrix, the poorer will be the convergence rate, which will mean having to increase the number of snapshots.

In the next section, some simulations examples are presented showing the variations of the number of adaptation cycles with respect to m, the SNR for both methods.

4. Simulations

We assume a uniform linear array with half-wave-length spacing containing m sensors; the number of signals is fixed to $d=5$ impinging from respective DOA's -40°, -20°, 28°, 30° and 60°. The signals denoted $s_i(t)$ for $i=1,...,5$ are assumed uncorrelated with respective power $\hat{s}_i$ and the additive noise is assumed white with power σ^2.

Several scenarios will be assumed, in each one the noise-subspace is estimated with algorithms implemented in the software package Matlab 4.2c.1, then a search procedure for the minimae of the estimated MUSIC spectrum is initiated using the Matlab fmin.m function which is a uni-dimensional minimization routine based on a golden section search and parabolic interpolation. In each simulation, we generate Monte-Carlo-type experiments consisting of 100 realizations.

Following the notation given in [2] and for ease of discussion, the first and the second proposed approaches are named respectively B1 and B2 as compared to the direct noise-subspace estimates A2 and A3 used in [2].

We assume in this case a stationary environment with equi-powered ($s_i(t)=1\ \forall i$) signals impinging from respective fixed DOA's at -40°, -20°, 28°, 30° and 60°. The signals and the noise are randomly Gaussian generated in Matlab with respectively unit-variance, and a fixed variance defined by the inverse SNR for the noise. The probability of detecting the signals at 28° and 30° will measure the performance of the algorithms. In fact, when the algorithms failed, due to the closeness of the two DOA's, only a single minimum was detected, approximately in the middle way between the two DOA's. In the case of correct detection, there were always two minimae at the actual DOA's, thus it was sufficient to look only at outcomes for the DOA at 28°. In all simulations, the other DOA's were always correctly detected. In the first experiment, we fixed SNR$=15\ dB$, varied the number of sensors from 10 to 40, and measured the minimum number of snapshots N for which the probability of correct detection was over 95 %. We estimate the values of N using the following simple simulation technique. For a fixed value of the given parameter (m or SNR), we randomly generate 100 sets of data according to the model shown in equation (1) and (3) with N

snapshots, then measure the probability of correct detection. We increase N, and the value for which this probability exceeded 95 % is the resulting estimate of minimum number of necessary snapshots.

The following table, corresponding to Fig. 1 was obtained:

m	10	15	20	25	30	35	40
N	350	60	30	18	12	10	8

Next, for a fixed $m=40$ and an SNR varying from 3 dB to 50 dB; the following number of snapshots with the same probability of correct detection are obtained (see Fig. 2):

SNR (dB)	3	7	11	15	20	30	50
N	22	15	11	8	7	6	5

A direct eigendecomposition of the sample covariance matrix under the same constraints on the same number of snapshots and values of m and SNR resulted in a MUSIC spectrum with the same probabilities of correct detection. Consequently, the first adaptive approach B1, having similar high-resolution capabilities, at least in the simulation examples could be compared to it in terms of computational complexity.

Repeating the same simulation under the same constraints for a direct noise-subspace adaptive solution using the gradient estimator ∇_1 of the adaptive algorithm A2 of [2] result in the following data. For SNR$=15$ dB (Fig. 1):

m	20	25	30	35	40
N	125	70	50	45	40

For m=40 (Fig. 2):

SNR (dB)	3	7	15	20	30	50
N	35	35	40	40	45	45

The number of snapshots actually increases as the SNR increases and this is consistent with the previous explanations since for higher SNR, the noise eigenvalues $\lambda_{d+1},\cdots,\lambda_m$ tend to get closer to the noise power, smaller compared to the signal ones, resulting in larger numerical errors as compared to method B1 above. Moreover, the actual number of operations for the method A2, as shown in table I in [2], is $\left(\frac{3m^2}{4}+\frac{m}{2}+m^2(m-d)+m(m-d)^2\right)N$. For the case when m-d>>d, clearly, A2 requires more operations than the proposed B1.

For the second approach B2, for a fixed SNR$=15$ dB, we obtain the data (shown also in Fig. 1):

m	10	15	20	25	30	35	40
N	2000	600	150	90	50	40	35

and for a fixed $m=40$ (Fig.2):

SNR (dB)	3	7	11	15	20	30	50
N	95	50	38	35	34	33	32

Those values could be compared to the LMS-type with the gradient estimator ∇_2 direct noise-subspace adaptive algorithm A3 of [2] for which the following tables are obtained. We point out that in this case, setting the normalizing matrix $\mathbf{M}(k)$ in equation (10) to the identity matrix leads to better convergence especially for higher SNR because the elements of the matrix $\mathbf{M}(k)$ can not be estimated with accuracy due to their direct relationship to the smaller eigenvalues [2]. For SNR=15 dB (Fig. 1):

m	20	25	30	35	40
N	400	250	140	100	90

For m=40 (Fig. 2):

SNR (dB)	3	7	11	15	20	30	50
N	130	135	100	90	85	80	80

Figure 3 and Figure 4 show plots of the approximate estimated computational costs for the different approaches, when m or SNR varies, respectively for SNR=15 dB and m=40. Clearly, the indirect signal subspace estimate approach, not only result in faster estimates as it requires less snapshots but also each update is less costly because of the savings in computations for this case (m-d<<d). Method B2 seems to result in fewer operations for m>25 when SNR=15 dB in Fig.3; B1 outperforms a direct eigendecomposition approach for m=40 and SNR greater or equal to 15 dB (Fig. 4), otherwise if SNR=15 dB then the number of sensors has to exceed 40 as shown in Fig. 3. For both methods A2 and especially A3 of [2], which could not be shown in both Fig. 3 and 4 because of an exceedingly large number of required operations, a direct eigendecomposition is always preferable given the particular assumptions on the simulation example.

5. Proposed beamforming technique

As mentioned in the introduction, the knowledge of the DOA's permit us to do efficient beamforming. In what follows, we propose a simple beamforming scheme where the DOA of the desired signal as well as those of the interferences are known. The previous adaptive DOA tracking could be used for an estimate of the impinging DOA's; additional information is though required to decide on which DOA is the desired or the interference one. We assume that the array response in equation (2) is known for all possible values of the DOA parameters $\theta = \begin{pmatrix}\theta_1 & \cdots & \theta_d\end{pmatrix}$ and also that θ_d is the desired DOA and the others are interferences. We arrange the array response $\mathbf{A}(\theta) = \begin{pmatrix}\mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_d)\end{pmatrix}$ in such a way that the desired array response to DOA θ_d correspond to the last column of $\mathbf{A}(\theta)$ then perform a QR factorization on $\mathbf{A}(\theta) = \begin{pmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{pmatrix}\begin{pmatrix}\mathbf{R}_1 \\ \mathbf{0}\end{pmatrix} = \mathbf{Q}_1\mathbf{R}_1$ where $\begin{pmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{pmatrix}$ is unitary with $\mathbf{Q}_1$ being of dimension m by d, $\mathbf{Q}_2$ m by m-d, $\mathbf{R}_1$ is a d by d upper triangular matrix and $\mathbf{0}$ is the m-d by d zero matrix. From equation (1), it can be readily derived that:

$$\mathbf{R}_1^{-1}\mathbf{Q}_1^{\dagger}\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{R}_1^{-1}\mathbf{Q}_1^{\dagger}\mathbf{n}(t) \tag{11}$$

The last row of $\mathbf{s}(t)$ corresponds to the desired signal and if we neglect the internal noise, it is equivalent to the last row of $\mathbf{R}_1^{-1}\mathbf{Q}_1^{\dagger}\mathbf{x}(t)$. However, the matrix $\mathbf{R}_1$ being upper triangular, so is its inverse $\mathbf{R}_1^{-1}$, therefore, if we let $\mathbf{w}^{\dagger}=\mathbf{q}_d^{\dagger}$ where $\mathbf{q}_d^{\dagger}$ is the last row of $\mathbf{Q}_1^{\dagger}$ (or $\mathbf{q}_d$ the last column $\mathbf{Q}_1$) and $\mathbf{w}^{\dagger}$ being the proposed beamforming coefficient vector, we get: $\mathbf{w}^{\dagger}\mathbf{x}(t)\approx ks_d(t)$ where $k=(\mathbf{R}_1)_{dd}$ the last diagonal element of the matrix $\mathbf{R}_1$ and $s_d(t)$ is the desired signal. Since the whole QR factorization on $\mathbf{A}(\theta)$ takes only an order of md^2 operations [11], it is comparable to the adaptive eigenspace estimation computational load and appears an attractive beamforming option. The count, actually is much less since only a single vector of the $\mathbf{Q}_1$ matrix is needed for this case.

We performed a computer simulation to compare the performance of the proposed beamforming approach to the reference signal method where the reference signal is assumed perfect (i.e. $s_d(t)$ itself). The DOA's estimates outputs from the previous adaptive algorithm B2 in the same communications scenario were taken to feed the proposed beamformer. Figures 5 show the ratio between the output Signal to Interference plus Noise Ratio (OSNIR) and the input Signal to Interference plus Noise Ratio (ISNIR) for the two different methods for an SNR of 3 dB for different values of the number of sensors. OSNIR and ISNIR are defined respectively as:

$$OSINR=\frac{E\left\{\left|\mathbf{w}^{\dagger}\mathbf{a}(\theta_5)s_5(t)\right|^2\right\}}{\sigma^2|\mathbf{w}|^2+\sum_{k=1}^{4}E\left\{\left|\mathbf{w}^{\dagger}\mathbf{a}(\theta_k)s_k(t)\right|^2\right\}} \text{ and } ISINR=\frac{E\left\{\left|a_1(\theta_5)s_5(t)\right|^2\right\}}{\sigma^2+\sum_{k=1}^{\hat{d}}E\left\{\left|a_1(\theta_k)s_k(t)\right|^2\right\}}$$

$E\{.\}$ denotes the statistical expected value and $\mathbf{w}^{\dagger}$ the corresponding steering vector. The difference between these two quantities represent the improvement in the signal estimation procedures by using adaptive beamforming.

We see that the performances are comparable and the proposed method could be used due to its simplicity for high-resolution DOA estimation followed by signal estimation.

6. Conclusion

Two adaptive algorithms for the determination of the noise-subspace of the MUSIC spectrum having some potential for a real-time implementation in sensor arrays, were derived. It was shown that this spectrum, characterized with known high-resolution capabilities to distinguish between two closely spaced DOA's, can be determined in a straightforward manner from the adaptive estimation of the signal-subspace requiring no additional computations. It was suggested that in situations of moderate to high SNR (> 15 dB), where the signal eigenvalues are much higher than the noise eigenvalues, it is numerically more efficient and faster to estimate the signal-subspace rather than the noise-subspace, especially in cases where the number of impinging signals is much less than the number of sensors.

Some approximate computational costs in terms of the necessary multiplications were shown for both approaches and compared with the conventional direct noise-subspace adaptive algorithms. The performance of the proposed algorithms was verified through computer simulations for a simple sensor array communications system and compared to the traditional adaptive solutions. Under the constraints on the SNR and the ratio between the number of sensors and the number of signals with appreciable level of power, conditions that are not too restrictive, the proposed solutions clearly lead to a significant decrease of the number of calculations.

In addition, although the adaptive approaches are more suitable to a real non-stationary communications environment, sometimes the number of snapshots necessary for the convergence to a sufficiently close MUSIC spectrum can become prohibitively so large that a direct eigendecomposition on the sample covariance matrix could be as well used. This has been demonstrated in the computer simulations for the provided examples especially for a low number of sensors; for a number of sensors larger than some threshold value which will depend on the particular communications system used in practice, the adaptive update approach could become attractive.

Finally, a simple beamforming technique relying on the DOA's knowledge was derived. Its cost in terms of the computations needed to derive the beamformer coefficients was shown to be sufficiently low as compared to the previous adaptive tracking algorithms and its performance superior to that of a reference signal method.

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References

[1] B.D. Van Veen and K.M. Buckley, "Beamforming: A Versatile Approach to Spatial Filtering", IEEE ASSP Mag., pp.4-24, April 1988.

[2] Yang, J. and Kaveh, M., "Adaptive Eigensubspace Algorithms for Direction or Frequency Estimation and Tracking," IEEE Trans. on Acous., Speech and Sig. Proc., Vol. 36, No. 2, pp. 241-251, Feb. 1988.

[3] Thompson, P., "An Adaptive Spectral Analysis Technique for Unbiased Frequency Estimation in the Presence of White Noise", Proc. 13th Asilomar Conf., pp. 529-533, Nov. 1979.

[4] Fu, Z. and Dowling, E.M., "Conjugate Gradient Eigenstructure Tracking for Adaptive Spectral Estimation", IEEE Trans. on Acous., Speech and Sig. Proc., Vol. 43, No. 5, pp. 1151-1160, May 1995.

[5] G.W. Stewart, "An Updating Algorithm for Subspace Tracking", IEEE Trans. on Sig. Processing, Vol.40, June 1992.

[6] Bellanger, M.G., "Adaptive Digital Filters and Signal Analysis", Marcel Dekker, Inc., New York and Basel, 1987.

[7] N.L. Owsley, "Systolic Array Adaptive Beamforming", Naval Underwater Sys. Center, Tech. Report 7981, Sept. 1987.

[8] Schmidt, R.O., "Multiple Emitter Location and Signal Parameter Estimation", Proc. RADC Spectrum Estimation Workshop, pp. 243-258, Rome, NY, 1979.

[9] Roy, R.H. and Kailath, T., "ESPRIT, Estimation of Signal Parameters via Rotational Invariance Techniques", IEEE Trans. on Acous., Speech and Sig. Proc., Vol. 37, No. 7, pp. 984-995, July 1989.

[10] M. Viberg, B. Ottersten and T. Kailath, "Detection and Estimation in Sensor Arrays Using Weighted Subspace Fitting", IEEE Trans. on Sig. Processing, Vol.39, No.11, Nov. 1991.

[11] W.H. Press, S.K. Teukolsky, W.T. Vetterling, B.P. Flannery, "Numerical recipes in C", Cambridge University Press, Sec. Edition, 1992.

List of figures

```
%Software listings used in the simulations to determine the minimal number of
%snapshots needed in the different adaptive algorithms for less than 5% detection
%failure DOA_est_beam.m
clear
%Set the number of impinging signals d
d=5;                                %Fixing number of signals
teta=[28 30 -20 -40 60]*pi/180;     %Fixing values of DOA's
m=input('Number of sensors');       %Set the number of sensors m
SNR=input('SNR');                   %Set the SNR in dB
gain=1/10^(SNR/20);
num_exp=100;                        %Set the number of experiments
id=eye(m);
mu=2/3/m;                           %Set the step-size for the LMS-type algorithms
randn('seed',87634);                %Set the Gaussian random generator seed
rand('seed',8764);                  %Set the uniform random generator seed
%Function computing the array manifold given m, d and DOA's
aa=arr_man(teta,m,d);
snap_min=input('Minimal number of snapshots');
snap_max=input('Maximal number of snapshots');count=0;
meth_choi=menu('Choose which algorithm','B1','B2','A2','A3');
for N=snap_min:snap_max
        count=count+1;
        cor_det=0;                  %Initialize the number of correct detection
        for lex=1:num_exp
%Randomly generate the signals and noise
                s=randn(d,N)*exp(j*(2*rand(N)-ones(N))*pi);
                nn=gain*randn(m,N)*exp(j*(2*rand(N)-ones(N))*pi);
%Generate the data according to the ULA model
                x=aa*s+nn;
%Initialiaze covariance matrix and signal or noise-subspace
                noi_sub=id(:,1:m-d);sig_sub=id(:,1:d);auto_cov=zeros(m);
                for l=1:N
%Adaptive algorithm B1 or B2 or A2 or A3
                        if meth_choi==1
                                b1;     %Method B1
                        end
                        if meth_choi==2
                                b2;     % Method B2
                        end
                        if meth_choi==3
                                a2;     % Method A2
                        end
                        if meth_choi==4
                                a3;     % Method A3
                        end
                end
%Routine to compute the approximate MUSIC spectrum then find the minimae
                mus_min;
        end
        res(count)=cor_det;         %Probability of correct detection
end
```

%Function routine to compute the array manifold of a uniform linear array with
%m sensors and half-wavelength spacing *arr_man.m.*

```
function am=arr_man(angl,m,dsig)
for ll=1:m
        for lll=1:dsig
                am(ll,lll)=exp(-j*(ll-1)*pi*sin(angl(lll)));
        end
end
```

```
%Routine for the simulation of method B1 b1.m.
for l=1:N
%Define the forgetting factor
        alpha=1/l;
%Update the covariance matrix
        auto_cov=(1-alpha)*auto_cov+x(:,l)*x(:,l)'*alpha;
%Update the signal-subspace
        v=auto_cov*sig_sub;
%Orthogonolize the signal-subspace (with the QR factorization)
        [q,r]=qr(v);
        sig_sub=q(:,1:d)
end
```

```
%Routine for the simulation of method B2 b2.m.
for l=1:N
%Update the covariance matrix
	dummy=x(:,l)'*sig_sub;
	sca_mat=mu*inv(diag(abs(dummy/dummy(d))));
%Update the signal-subspace
	v=sig_sub+x(:,l)*dummy*sca_mat;
%Orthogonolize the signal-subspace (with the QR factorization)
	[uq,r]=qr(v);
	sig_sub=uq(:,1:d);
end
```

```
%Routine for the simulation of method A2 a2.m.
for l=1:N
%Define the forgetting factor
        alpha=1/l;
%Update the covariance matrix
        auto_cov=(1-alpha)*auto_cov+x(:,l)*x(:,l)'*alpha;
%Update the signal-subspace
        v=noi_sub-mu*auto_cov*noi_sub;
%Orthogonolize the signal-subspace (with the QR factorization)
        [q,r]=qr(v);
        noi_sub=q(:,1:m-d);
end
```

```
%Routine for the simulation of method A3 a3.m.
for l=1:N
%Update the covariance matrix
        dummy=x(:,l)'*noi_sub;
        sca_mat=mu*inv(diag(abs(dummy/dummy(d))));
%Update the signal-subspace
        v=noi_sub-x(:,l)*dummy*sca_mat;
%Orthogonolize the signal-subspace (with the QR factorization)
        [uq,r]=qr(v);
        noi_sub=uq(:,1:m-d);
end
```

%Routine to compute the approximate MUSIC spectrum then search its minimae
mus_min.m.

```
if meth_choi==1 | meth_choi==2
        music_spec=eye(m)-sig_sub*sig_sub';
else
        music_spec=noi_sub*noi_sub'
end
%Minimize the MUSIC spectrum around the angle intervals [26, 29] and [29, 32]
%passed to the matlab function fmin.m where the string 'f' contains the name of
%the objective function to be minimized (maximum number of iterations 20)
[dummy(1) o1]=fmin('f',26,29,[0 .0001 .0001 0 0 0 0 0 0 0 0 0 0 20],music_spec,m);
[dummy(2) o2]=fmin('f',29,32,[0 .0001 .0001 0 0 0 0 0 0 0 0 0 0 20],music_spec,m);
if o1(10)<21 & o2(10)<21
        cor_det=cor_det+1;              %Increment number of correct detection
end
if input('Do you want to beamform ?')=='y'
        beam_form
end
```

%'f' is a string containing the name of the objective function to be minimized f.m.

```
function [q, options]=f(tet,p,m)
a=arrmana(tet*pi/180,m,1,0);
q=real(a'*p*a);
```

```
%Routine to simulate the proposed beamforming method compared to a reference
%signal method beam_form.m.
%Estimated DOA's (arranged in an array such that the desired signal is its last
%element
teta=input('Enter estimated DOA''s (in degrees) desired DOA last element')*pi/180;
%Compute the array manifold according to the estimated DOA's.
aa=arr_man(teta,m,d);
%Find the beamformer elements for the proposed approach w1 and the reference
signal method w2
[q,r]=qr(aa);
w1=q1(:,d)';
rx=x*x'/N;                          %Sample covariance matrix
rd=x*s(5,:)'/N;                     %Sample cross-correlation matrix
w2=inv(rx)*rd;
w=input('Which beamforming method w1 or w2');
%Compute the Output and the Input Signal to Interference plus Noise Ratio
for inc=1:d
        sig_aft_beam(inc)=abs(w'*aa(:,inc)*s(inc,:)).^2;
        sig_bef_beam(inc)=abs(aa(:,inc)*s(inc,:)).^2;
end
osinr=sig_aft_beam(d)/(sum(sig_aft_beam(1:d-1))+g^2*w'*w);
isinr=sig_bef_beam(d)/(sum(sig_bef_beam(1:d-1))+g^2);
```

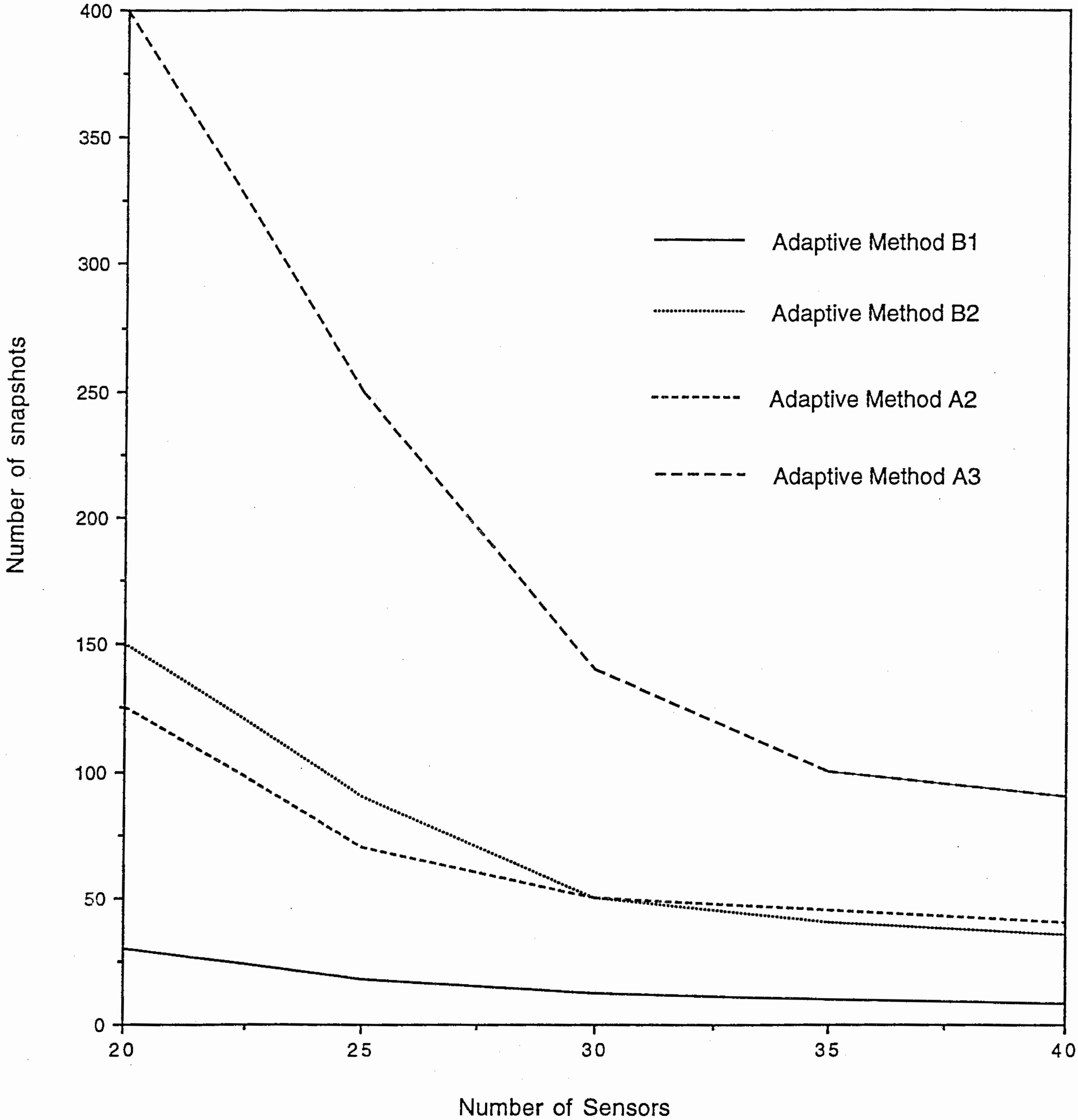

Figure 1 Number of needed snapshots for the different adaptive methods versus the number of sensors SNR=15 dB.

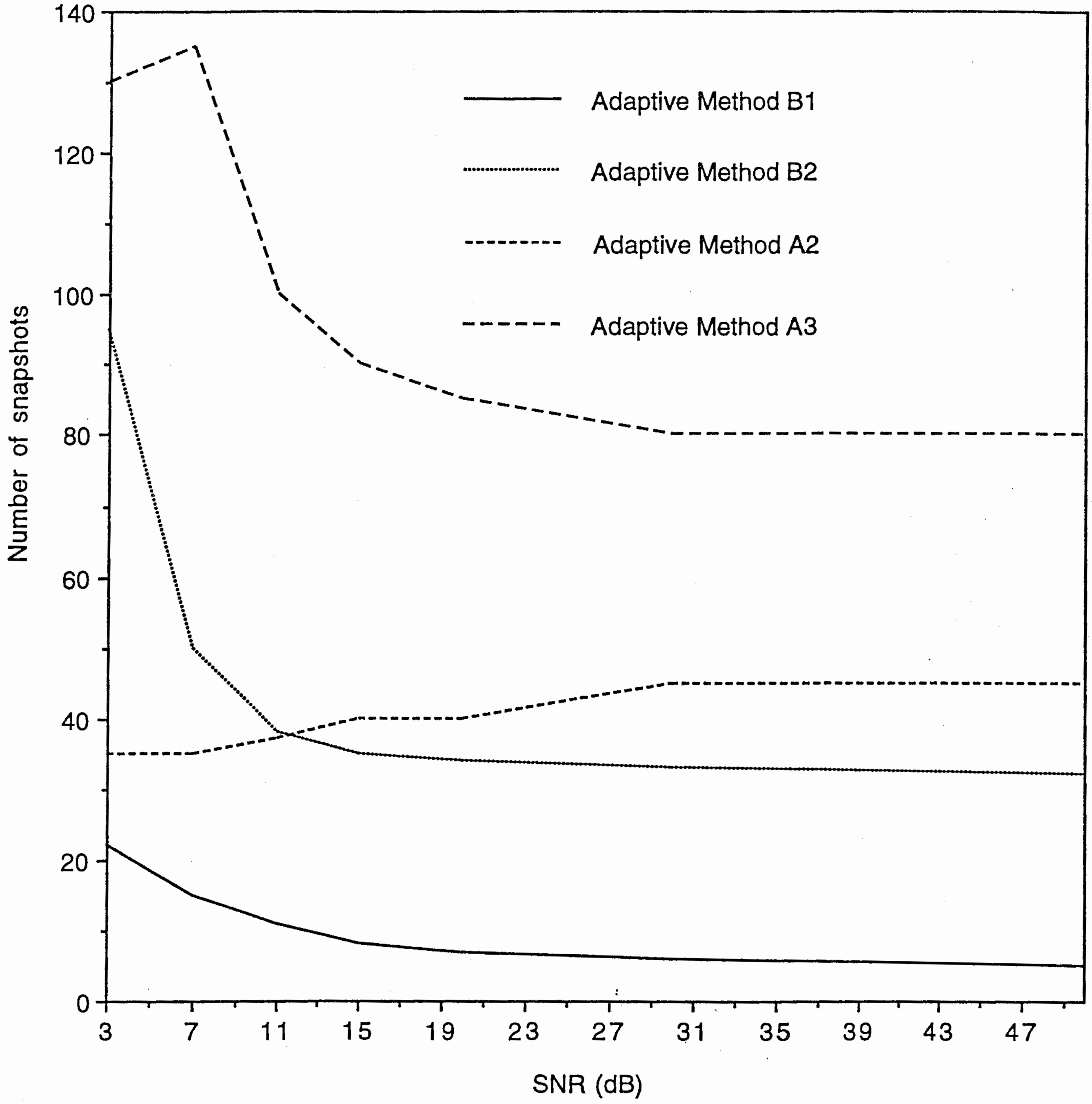

Figure 2: Number of needed snapshots for the different adaptive methods versus SNR (in dB) for m=40.

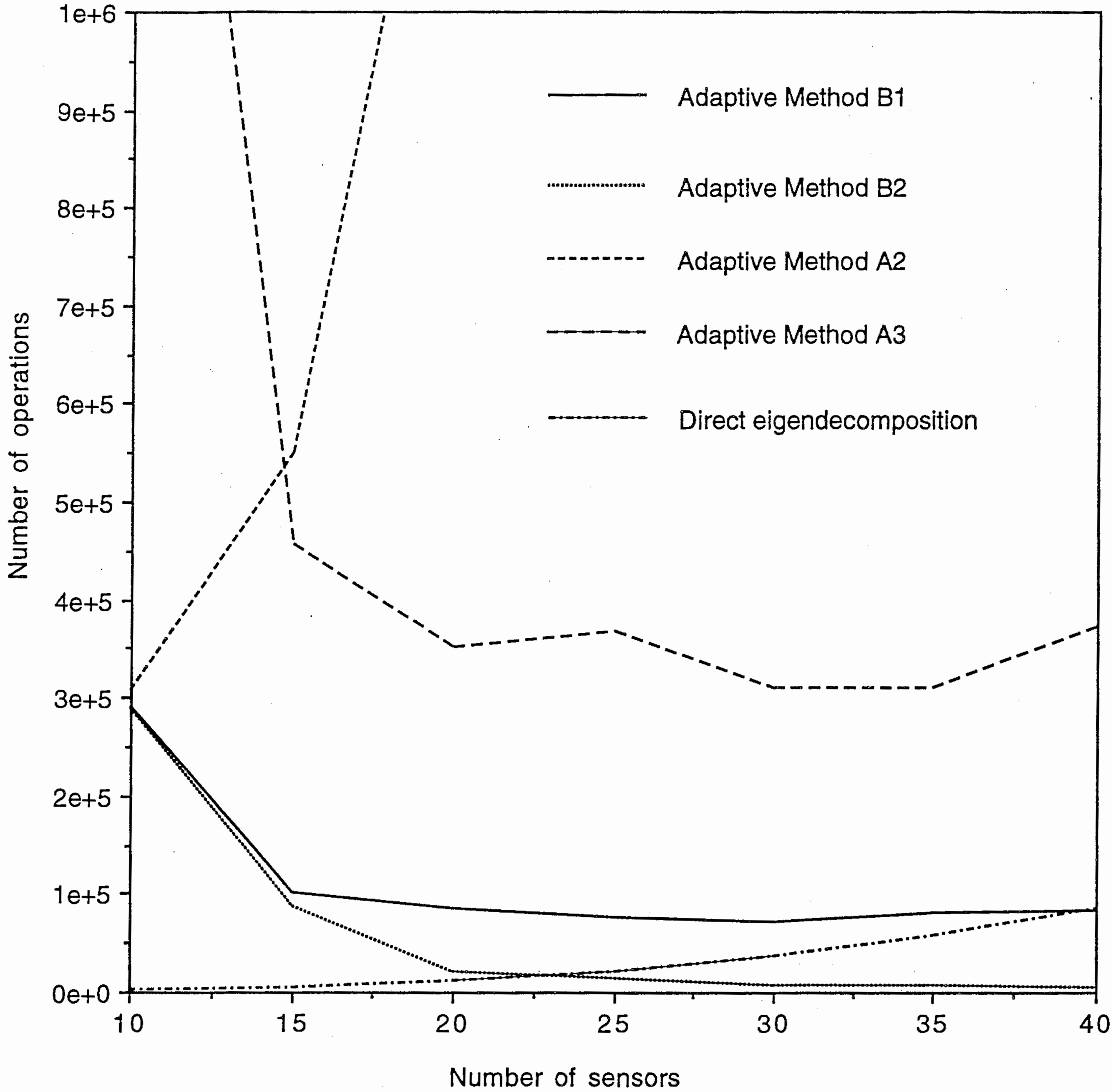

Figure 3: Total number of operations for different DOA estimation techniques versus the number of sensors for SNR=15 dB.

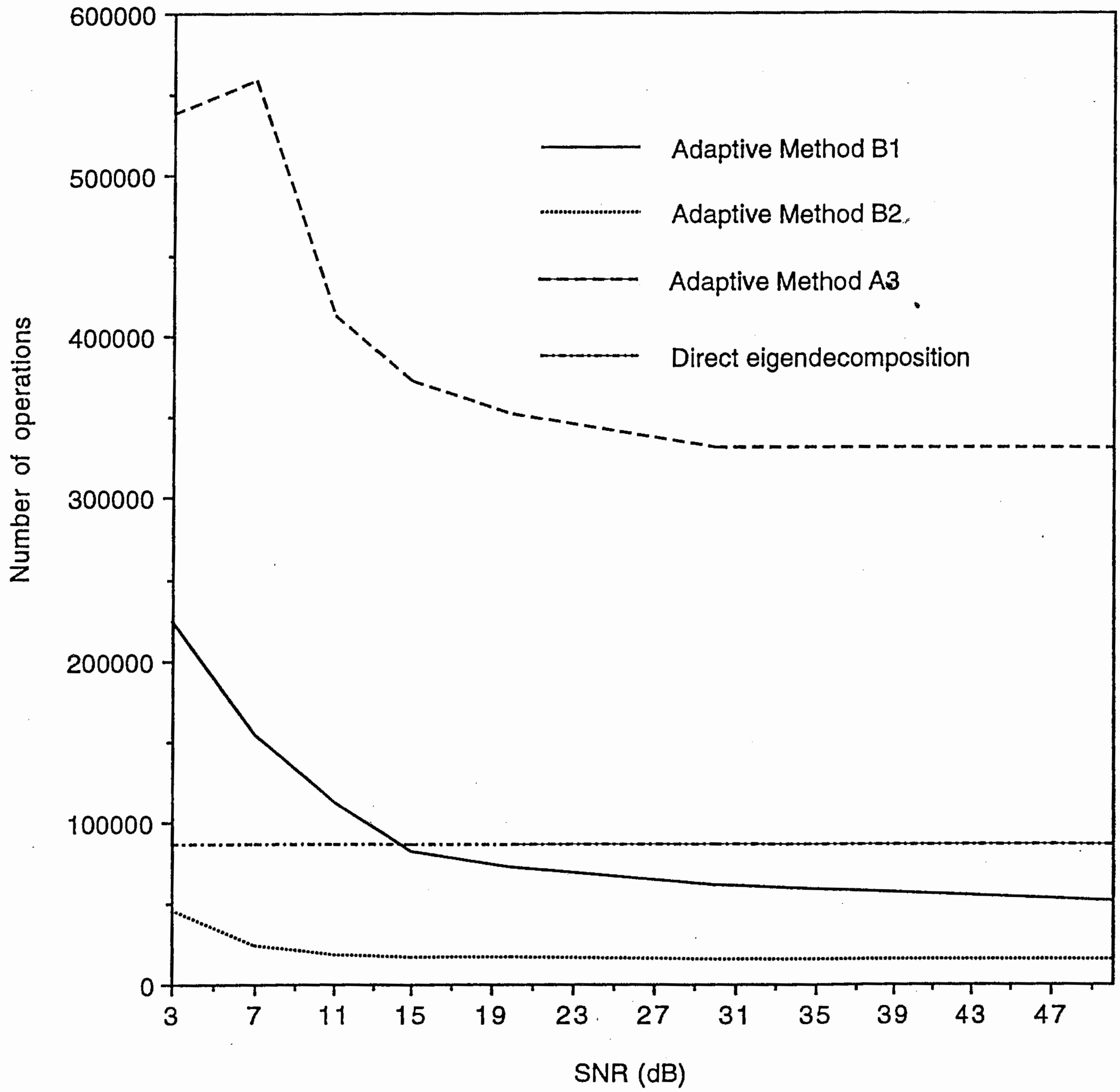

Figure 4: Total number of operations for different DOA estimation techniques versus SNR (in dB) for m=40.

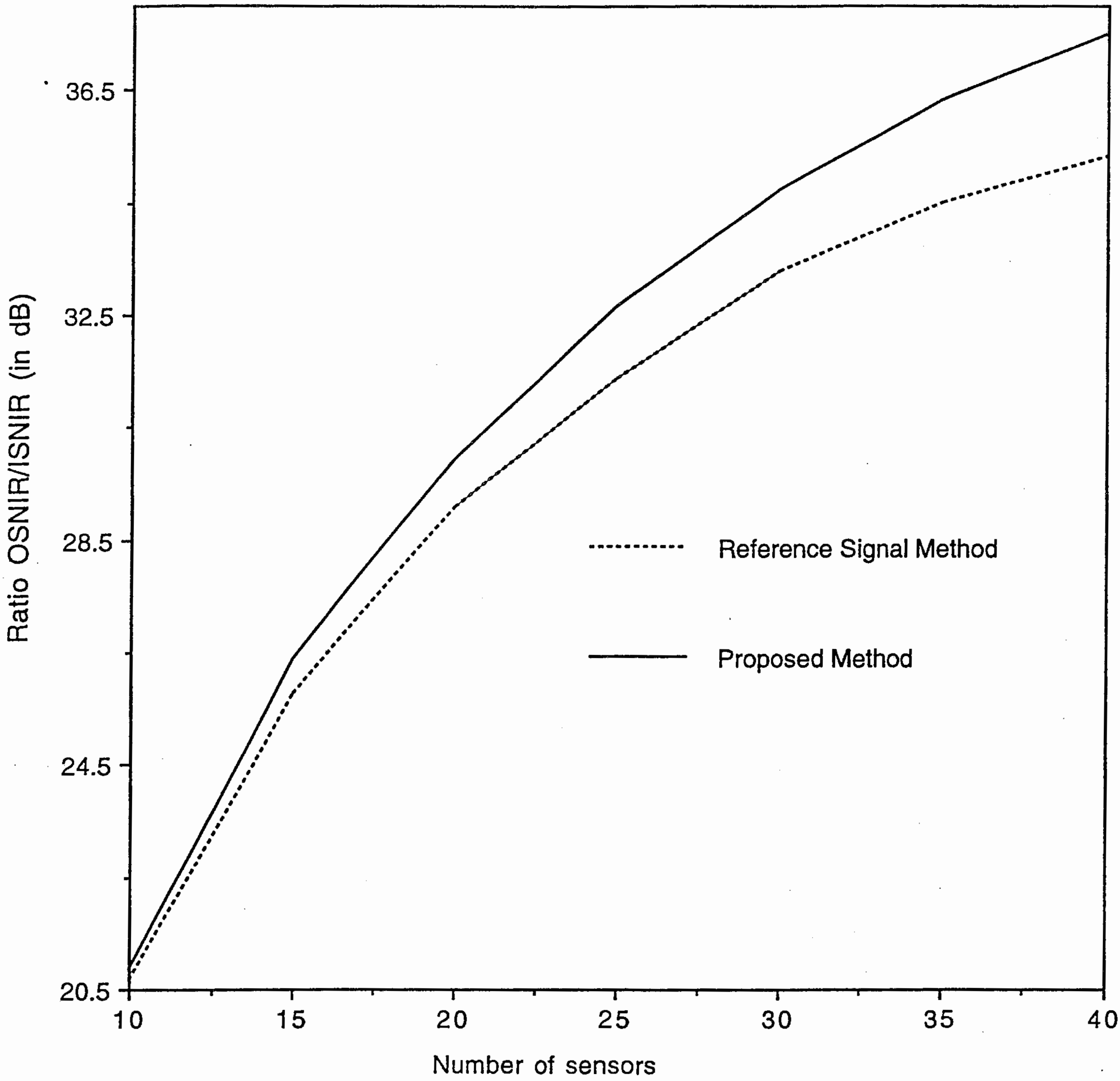

Figure 5: Performance of the proposed beamforming method compared to the reference signal method for an SNR of 15 dB.