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Towards experiments in functional optical chaos

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Abstract

This report reviews certain delayed-feedback type optical systems with which it should be possible to demonstrate control principles for functional chaos. It is proposed that two complementary methods for switching between complex multistable oscillation modes, Seeded Bifurcation Switching ("SBS") and Chaotic Switching ("CS"), would be productive areas of investigation.

Contents

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T	Introduction	2
2	Delayed-feedback system (DFS) Theory2.1 theoretical model2.2 oscillations in delayed-feedback system	4 4 4
3	Control Principles: Codes and Switching3.1 introduction	5 5 6 7
4	DFS Hardware4.1 introduction4.2 electrooptic hybrid DFS4.3 all-optic DFS	8 8 8 10
5	Hardware for Switch Controls	11
6	DFS Networks	12
7	Concluding remarks	12

1 Introduction

Chaos is known to occur commonly in nature, and in particular in biological systems where it has been conjectured to be of use for complex information processing and control. However, there have been fundamental gaps in the understanding of the role of chaos in biological systems, and its possible applications in artificial systems. We support the view (figure 1) that chaos, as complex dynamics from simple rules, can make possible complex functions in simple devices. However, fundamental control principles need to be found to establish the functional role of chaos. We are trying to establish such fundamental principles. In particular we are considering Input/Output algorithms for nonlinear optical systems which will provide the basis for applications of complex dynamics in optical signal processing and computing. Figure 2 shows a block diagram of the general sort of scheme we are considering in which a nonlinear optical system is controlled or "accessed" via an interface consisting of an input processor and output processor.

For a number of reasons we have concentrated recently on the delayed-feedback system (DFS) illustrated in figure 3. The DFS system can exhibit a large variety of self-oscillation behaviour. Depending on the value of the constant input intensity and the initial optical signal in the system, the output intensity can be oscillate in any one of a large number of different oscillation patterns. The fundamental period of oscillation is proportional to the delay-time. This phenomenon of "delay-induced" or "Ikeda" oscillations in delayed-feedback systems is now quite well known. A text by Gibbs contains a useful introductory review from the viewpoint of optical systems. Investigations of DFS were pioneered theoretically and numerically by K. Ikeda at the Department of Physics, Kyoto University and experimentally by H. Gibbs at the Arizona University Optical Sciences Center.

Figure 5 shows schematically how the oscillation modes change as a parameter such as the input intensity is varied. The modes form a tree with respect to increase of the parameter. At each branch (bifurcation) point a mode becomes unstable and new modes appear. The existence of multiple branches for a given parameter means multistability - which mode is realized depends on the initial signal. As bifurcation progresses the number of modes exponentially increases, as does the complexity of the oscillation patterns. Eventually, the branches start to overlap or merge. Here the oscillations are chaotic, or "turbulent". At first the fluctuations are relatively small, but as branch merging progresses with the increase in parameter the oscillations become more and more turbulent.

The main reasons for our interest in the delayed-feedback system are the delayed-feedback system is quite general with bifurcation structure which is relatively orderly, and in principle, should be simple to use for generating optical signals which are easy to code and decode. We are particularly interested in the intermediate range, from just before chaos where there is a large number of stable oscillation modes ("prechaos"), and the stages of branch merging upto the onset of fully developed chaos.

We have succeeded in showing how it is possible in principle to selectively switch between oscillations in a delayed-feedback system. The two switch methods we proposed are (I) Seeded Bifurcation Switching (SBS), and (II) Chaotic Switching (CS) The former allows "direct access" or deterministic switching of stable signals corresponding to a large number of bits of information, and makes practically possible the systematic access of prechaos. The latter allows adaptive "search access" of oscillations and is, we believe, the first concrete example of the usefulness of the bifurcation of chaos itself.

These are fundamentally new ideas and although tested successfully in computer simulations of model systems, they have not yet been tried in any physical system. Thus, it is important to experimentally test (a) the fundamental techniques of switching, and (b) devices and systems with practical applications.

Our objective in this report is to consider how specific schemes for (a) may be realized. In future reports, we plan to propose system configurations for applications to tasks of optical signal and information processing. As our specific objective is to assist planning for experimental work by filling in details which are not documented elsewhere, we shall for brevity, not go into details which are available in the references. This report should be read in conjunction with the references. First we briefly review the dynamical principles of hierarchically bifurcating oscillations in a delayed-feedback system (DFS). In the next section we review our two switching methods. Then we discuss physical realizations of the DFS system, and related issues.

3

2 Delayed-feedback system (DFS) Theory

2.1 theoretical model

The DFS can be realized in many different types of systems. The essential features of the optical DFS considered here are a certain type of nonlinear dependence of intensity I on phase shift x, and a dependence of the phase shift in turn on intensity fedback with delay T_r . Although the intensity modulation part and feedback part need not necessarily be physically distinct (see for example Nakatsuka et al.), for simplicity we shall consider the delayed-feedback scheme shown in figure 3c consisting of three physically distinct components - a constant intensity source, a modulator and a feedback path.

The essential dynamics of the DFS system are described by the following equation of motion

$$T_m dx/dt = -x + x_0 + \mu I(x(t - T_r))$$
(1)

 x_0 is a constant phase shift term. The parameter μ is proportional to the intensity of the input light and the gain in the system. The output intensity is proportional to μI . The response T_m is determined from the time for relaxation of x after the feedback has been cutoff.

It is assumed that the dependence of I on x is such that the function

$$x_{n+1} = \mu I(x_n) \tag{2}$$

exhibits a type of bifurcation behaviour known as period-doubling bifurcations as μ is increased.

2.2 oscillations in delayed-feedback system

The connection between the period-doubling function in equation (2) and the solutions of equation 1 is demonstrated in figure 4. Here figure 4a shows the dynamics of the mapping (2). Figure 4b shows how a solution to (1) is constructed by mapping points separated by T_r . As parameter μ is increased, stable solutions of the map bifurcate from period 1, to period 2, to period 4 and so on, toward chaos. Figure 5 shows how there is a corresponding cascaded bifurcation of the stableoscillation solutions of equation (1). Since the separation of periodic points of the map decreases exponentially with the bifurcation order m, solutions

4

can be thought of as nth harmonic carrier oscillations, period about $2T_r/n$, appearing at the first bifurcation m=1, and increasingly higher order modulations appearing with successive bifurcations. In figure 5b are shown particular examples of (n=3,m=2) solutions, where n is harmonic number and m is the bifurcation order.

3 Control Principles: Codes and Switching

3.1 introduction

Our essential idea is that the DFS is an optical signal generator which can generate a great variety of signals. Signals generated by the DFS can be used for handling information if (1) we can code the signals, and (2) we have some method of taking input information and using it to switch selectively between ouput signals. Figure 6 summarizes the two switch methods we have proposed for DFS.

3.2 coding and capacity for prechaos

There are many conceivable ways of making the correspondence betweeen DFS oscillations and input/output information. Figure 7 shows a useful decoding scheme which uses threshold devices to convert the DFS signal to a set of secondary pulse signals. In our first switching method, the direct access a pulse signal will be treated as a sequence of binary bits.

How many bits of information can we handle using one set of (n,m) modes? If the number of stable oscillation patterns which can be switched is M, the information capacity in binary bits is $N = log_2 M$. We can think of a (n,m)mode as n temporal bits - with each interval of length T_r/n corresponding to one temporal bit. By associating a shift up with "1" and a shift down with "0" at each bifurcation we can assign an (m-1) bit binary word to each temporal bit. For a given n and m the capacity is $n \times (m-1)$ bits, assuming we have a reference clock to distinguish temporal patterns translated along the time axis.

There is an intrinsic upper bound on n proportional to effective delay $t_{\tau} = T_{\tau}/T_m$, and a practical upper bound on accessible m determined by the Signal-to-Noise Ratio (SNR), so that the capacity N is bounded, with the bound for

large effective delay given by

$$N = .13(T_r/T_m)log(SNR+1)$$

(3)

3.3 seeded bifurcation switching (SBS)

The first switch method, is called Seeded Bifurcation Switching, or SBS for short. Details are found in "Bifurcation and Dynamical Memory" by Davis and Ikeda. In this method (figure 8), the input processor takes an input digital code signal and causes a switch to a corresponding oscillation by sending a combination of a bifurcation control signal to the parameter μ and a seed signal to the phase bias x_0 . The output processor can be used for monitoring the oscillations to synchronize switch signals, and for decoding the oscillation.

This switching method can be used in digital control optical signal generator and optical memory using nonlinear oscillations. (see Davis, "A method for ... ", 1988) It is also possible to perform logical operations by connecting delayed feedback oscillators in networks. The essence of the logical operations by seeded bifurcation switching is voting(majority) logic. For example if we use m=2 coding and the input to a DFS is a set of inputs from other DFS, for each temporal bit the switch is to "1" ("0")if the majority of the inputs in the time slot are "1"("0"). Logical "AND" is achieved by adding a bias bit of "0", and logical "OR" by adding a bias bit of "1". ¹ Logic for higher order bifurcations m > 2though possible in principle is rather more complicated and of dubious practicality at present.

3.4 chaotic switching (CS)

The second switch method is called Chaotic Switching, or CS for short. Details can be found in "Functional chaotic switch in a delayed-feedback model" by Davis. In this switch method, illustrated in figures 9 and 10, , the input processor regularly compares the code from the ouput processor with the input code and then adjusts the bifurcation control. The chaotic switch combines a tendency of

¹This voting logic is similar to that applied in the parametron device (Takahashi 1968), where switching is also in a sense bifurcation switching. A fundamental difference from the parametron idea is that here a single device is associated with many bits n. A parametron is a single temporal bit device. Even if we couple a delay loop with a parametron device the number of bifurcation parameter steps is equal to the bit number.

the chaos to cause wandering among many oscillation states with feedback on how well the states match an input code. Hierarchical bifurcation of chaos plays an essential role in coaxing the system into generating a signal which matches the input code. The method can be used simply to switch with compressed code or to do more sophisticated tasks involving "search", such as constraint satisfaction. Another exciting possibility is a type of "fuzzy" chaotic logic where some features are used for input and others for output. The DFS dynamical state constrained by a set of input features via the switch method is a type of dynamic membership function (possibly chaotic) corresponding to the set of input features.

3.5 DIRECT versus SEARCH

It is possible that the two methods could be used together. Consider the case of random switching in which a signal is to be transmitted to one of a number of next stages, but the destination is to determined dynamically on the basis of some criterion such as which destination is available. Using the m=2 pattern to correspond to the information to be transmitted and the m=3 pattern to correspond to the destination address, we could first use chaotic switch on the m=3 pattern and later SBS on the m=2 pattern.

The two switching methods are qualitatively different and they are complementary. SBS is direct access in which the destination and how to get there is specified exactly by the input. The CS is search access in which the destination is only specified to the extent that it satisfies certain criteria. DIRECT access is used when there is a direct and known correspondence between input and output. It is typically faster and more reliable than SEARCH access. SEARCH access makes access possible when there is not enough information for DIRECT access, and is thus related to the possibility of information generation.

Conventional information processing systems use DIRECT access at the fundamental hardware level, and introduce SEARCH access at a higher processing level using random number generators (with for example Monte Carlo algorithms). More recently the positive use of thermal noise in neural networks indicates how devices such as noisy neurons can be useful. We believe complex information processing can be achieved in simple devices by effectively using both DIRECT access and SEARCH access. Our work in the DFS system demonstrates how DIRECT access and SEARCH access can both be achieved in the intrinsic dynamics of a simple fundamental device. We are only just beginning to understand the implications and consider the possible applications of such a

7

device.

4 DFS Hardware

4.1 introduction

The delayed-feedback oscillation phenomena can be realized in a variety of optical systems. First we concentrate on hybrid electro-optic systems, because as a well-developed technology, they are most immediately practical for our two purposes. Later we will consider all-optic configurations which will be worth considering for applications in the future. We recommend Kaplan et al. for an earlier review of DFS in respect to generating (n=1,m=1) optical square wave signals.

4.2 electrooptic hybrid DFS

In an electrooptic realization of our system, we consider the modulator, and the input and output controls to be electrical. Figure 11 shows a typical configuration for a delayed-feedback signal generator.

There is feedback from the light intensity via a photodetector and DC amplifier to the voltage control of an electro-optic modulator. The equations relating the phase shift, x, inside the modulator medium with the applied voltage V, the input intensity I_{in} and the output intensity I for the electrooptic modulator are

$$x = CV \tag{4}$$

$$I = I_{in}(1 - \cos(x))/2$$
(5)

If we operate in the linear regime of the feedback the equation describing the change in phase is

$$T_m dx/dt = -x + x_0 + gI \tag{6}$$

The constant phase term is due to a bias voltage applied to the modulator. The gain factor g depends on the losses in the system, the fraction of light fedback and the gain of the DC amplifier. The composite response time T_m is determined as the time taken for the phase shift to relax when the feedback is cut off. The bifurcation parameter μ given by

$$\mu = g.I_{in}/2\tag{7}$$

is proportional to the system gain and the input light intensity.

In a hybrid system, the response time includes the characteristic response of the electro-optic effect, and the response time of the electricals (photodetector and amplifier). However the former can be of order nanoseconds and it is typically the latter which effectively determines the order of magnitude of the response time. Clearly the electricals will be faster for waveguide modulators where voltages can be smaller.

Figure 12 shows examples of electrooptic modulators obeying equations 4 and 5. The first in figure 12a is a nonlinear medium (such as a $LiNbO_3$ crystal) sandwiched by two polarizers which have their axes perpindicular to each other and at 45 degrees to the optical axes of the crystal (Gibbs et al., Okada et al.), so that

$$I = I_{in}(1 - \cos x)/2 \tag{8}$$

Figure 12b shows a waveguide device consisting of a MZ interferometer and a 3dB ($\lambda/4$) coupler (Haus). The intensity at the input I_{in} is related to intensities I_1 and I_2 at the respective output ports by

$$I_1 = I_{in}(1 - sinx)/2$$
(9)

$$I_2 = I_{in}(1 + sinx)/2 \tag{10}$$

An alternative to the coupler is a 4-port such as in figure 12c which is a junction between two symmetric guides and two asymmetric guides. (Izutsu and Sueta)

To access a variety of patterns (large information capacity) we need a large effective delay. The effective delay is the ratio of the actual delay time and the system response time. To increase effective delay we need to decrease reponse time, or increase delay time without similarly increasing response time.

In the first observation of bifurcation to chaos in a DFS (figure 13a, taken from Gibbs 1985), a group at University of Arizona used a computer to make an electronic delay of the order of 40 ms to realize large effective delay, $t_r = 40$, for electrical responses of order 1ms. Digitization noise limited T_m to the order of milliseconds. In a later experiment (figure 13b), the same group used a 1km optical fibre to achieve a similar effective delay with T_m of .33 to 1 μs .

A group at NHK (figure 14, taken from Okada and Takizawa 1981) used a coaxial cable electrical delay line to get a delay of microseconds and an effective delay of order one. As they couldn't increase the coaxial line length without also increasing response time, they could only observe the n=1 branch oscillations.

4.3 all-optic DFS

For faster signal processing we should consider all-optic systems. In all optic systems we can hope for response times of order pico- or even femto-seconds. In all-optic systems we let the feedback light itself modulate the refractive index, n, directly - for example in a Kerr medium where

$$T_m dn/dt = -n + n_2 I \tag{11}$$

The stationary nonlinear phase shift x is given by

$$x = (P/A_{eff}).(2\pi/\lambda).L.n_2$$
(12)

P power, A_{eff} effective area, λ wavelength, L interaction length, n_2 Kerr coefficient (cf. Nakatsuka et al. and Firth et al.) We need phase shifts x of order π . The PL product, the product of P and L necessary to achieve a π phase shift, is a useful measure for fibres and waveguides. (Note that $n_2 = \chi_3 c$, where $c = 12\pi/n_0$ in esu units, and $c = 3/(e_0.n_0)$ in mks units; $\chi_3(cgs) = 8.1 \times 10^{18} \chi_3(mks)$ and $n_2 esu = c^2 \times 10^{-8} n_2 mks$).

The first observation of all-optic delay-induced instability was done by a Kyoto University group using pulses in a single-mode fibre (figure 15, taken from Nakatsuka et al. 1983). There the source was a gas laser, an actively modelocked and Q-switched yttrium-aluminium-garnet laser, with pulse-peak power of 1kW. Delay-induced oscillation was observed as the modulation of a pulse sequence. The delay time, around the 1.2m fibre loop, of 7.6 ns corresponded to the pulse separation. n_2 of the core of the silica glass fibre used was quoted as 10^{-13} esu.

Recent advances in fibre technology should make it possible to realize delayinduced oscillations in short fibres at laser diode powers. PL products of the order 11Wm are now possible $(n_2 = 5.4 \times 10^{-20} m^2/W$ for GeO_2 doped core, White et al.). In the future, organic-core single mode fibres (Yamashita et al.) will be available with at least an order of magnitude higher nonlinearity.

It should be emphasized that all-optic scheme in figure 15 requires interference between the delayed-feedback light and the input light, so the light must have a coherence time considerable longer than the delay-time. An all-optic scheme closer to the style of the hybrid scheme which uses only the intensity (not the phase) of the feedback could be realized by adapting the scheme in figure 16 (taken from Kimura et al. 1986) devised by researchers at NTT for an all-optic

10

logical 'AND' gate This suits our purpose if we input our source light in one input and feedback the output to the second input (beam 2). The transmitted power P is

$$P = P_{in} \sin^2(2\theta) (1 - \cos x)/2 \tag{13}$$

Here P_{in} is the input power. x is the phase shift due to intensity-induced birefringence

$$x = 2\pi L \chi (P_x - P_y) / 3\lambda \tag{14}$$

 θ is the angle between the polarisation of the input and the principal axes of F_1 , P_x and P_y are the intensities of the input along the principal axes of F_1 . L is the fibre length and λ is the vacuum wavelength. χ is calculated using the self-focusing coefficient.

Recently there has been some attention on all-optic waveguide modulators (see Jensen for design principles, Lattes et al. for $LiNbO_3$, Li Kam Wa et al. for GaAs MQW). These use an intensity dependent refractive index to alter the phase matching condition between coupled channel waveguides, in a "nonlinear coherent coupler". (Figure 17). Although there are few examples of such systems at present, they seem to hold a lot of promise for the future.

5 Hardware for Switch Controls

In designing switch controls for the DFS we should keep in mind the fact that in method I, the switch controls need to be as fast as the signal modulation. In method II. they can be much slower. In regard to method I we should first test a single DFS as a signal generator and memory with electronic processor control. We could also consider logic with summing of signals at the photodetector. In future all-optic schemes we could consider control of the bifurcation parameter by optical switching of the source light intensity I_{in} instead of the electrical gain, and all optic seeding by refractive index modulation.

In regard to method II, at present electronic bifurcation control is practical until optical dynamics of rule selection are devised. The signal code match tests could be done optically without much difficulty.

Anyway, as a first step a hybrid experiment is desirable, and for a hybrid experiment straightforward electronic controls will be sufficient.

6 DFS Networks

In an all-optic configuration, the output intensity of one DFS is coupled directly to the next DFS so that it modulates the seed x_0 . DFS coupling is possible for small fanouts with waveguide or fibre coupling, and for larger fanouts with holographic connections.

Network configurations using DFS should combine spatially parallel processing with the temporal processing done by single DFS. Advanced information processing tasks could be achieved by combinations of SBS and chaotic switching at single device level and on various network levels. In this sense the role of a DFS should be compared with an active neuron. (Aihara)

DFS in networks could evolve independently, coupled only through the external switch control, or be dynamically coupled. Dynamic coupling between DFS could occur at discrete time intervals via SBS (with voting logic). In this case the network would operate similar to a neural network, with the added complexity of the delay pattern. Little work has been done on this aspect of neural networks. Alternatively, the DFS could be coupled continuously in time, as in the system shown in figure 18, in which case the dynamical behaviour can be expected to be complex spatio-temporal chaos. Again, no simulations of such a system have been performed to date, although numerical work on coupled map lattices (Kaneko 1989) are of some relevance. We shall leave further discussion of these network possibilities for a future report.

7 Concluding remarks

There are many nonlinear optical systems which exhibit delay induced and other types of chaos. The particular systems considered here are at this time the most practical for our two purposes - (a) testing the techniques of switching, and (b) thinking about immediate applications. We feel that tests of schemes in which bifurcation toward chaos and bifurcation of chaos itself play a definite and positive role are of fundamental significance, and could lead the way to many new applications of complex nonlinear phenomena.

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parameter μ

Figure 5a. Schematic bifurcation tree of stable oscillations in equation (1), labelled by harmonic number n and bifurcation order m



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Figure 6. Switch methods and their relevance to demonstrating the functionality of chaos in a complex dynamic optical signal generator



Output processor/Decoder

Figure 7. A general decoding scheme for the DFS used in the CS scheme and applicable in general to output processing of DFS signals. Secondary signal S_i has a pulse of width h_i when X_i exceeds a threshold value g_i , except if a threshold crossing comes within h_i of the previous crossing.



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Figure 8a. SBS control configuration and control rule



Figure 8b. SBS switching example







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generator (DCSG) and oscillation memory (OM) (cf. Patent application, Davis 1988). In /Out data is n binary bits. Algorithms for the seed and gain signal generators are obtained from the SBS algorithm in figure 8 together with equations 4 and 8.











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