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Abstract

The importance of geometric invariants to many machine vision tasks, such as model-based recognition, has been recognized since an object generically has its own value for an invariant. A number of recent studies on geometric invariants in a single view concentrate on coplanar objects: coplanar points, coplanar lines, coplanar points and lines, coplanar conics, etc. Therefore, it is essentially only to 2-D objects that we can apply a method using geometric invariants. This paper presents a study on geometric invariants of noncoplanar objects, i.e., 3-D objects. A new geometric invariant is derived from six lines on three planes in a single view. The distribution of the six lines is clarified and the condition under which the invariant is nonsingular is also described. In addition, we present some experimental results with real images and find that the values of the invariant over a number of viewpoints remain stable even for noisy images. We can apply a method using geometric invariants to 3-D objects as well.

Key Words: geometric invariant, noncoplanar lines, nonsingularity, 3-D object recognition.

Contents

1 Introduction

We human beings can easily recognize objects in 3-D through visual 2-D information. The appearance of an object shape projected onto the image plane depends on the position of the object relative to a viewpoint, which results in numerous different images, even for the same object. This complicates many problems in machine vision. Therefore, extracting the properties that remain invariant under any change in viewpoint should provide useful information. Geometric invariants are functions that are unaffected by a change in viewpoint and that are defined by coordinates of point images, or coefficients of equations that represent line (or curve) images. Accordingly, using geometric invariants leads to powerful methods [8], [13] of supporting a number of machine vision tasks such as object recognition. For instance, a classical approach to model-based object recognition is divided into two procedures: for a given image, 1) determine the position of an object relative to a viewpoint, i.e., *pose determination*; and then 2) compare the given image of an object with every image that is stored in a library of images to identify the object. However, if we use geometric invariants of an object, attaching the values of the invariants to images in the library makes it possible to directly compare the given image with one in the library without executing procedure 1), and, furthermore, allows a reduction in the number of images to be compared [3], [12]. As for the problem of model description, how to describe the shape of an object is the main concern. Using invariant shape descriptors is definitely more efficient since such descriptions are unaffected by a change in viewpoint. As has been seen, geometric invariants are not only important but are also readily applicable to problems in the field of computer vision.

From this point of view, the importance of invariants has been recognized since the origin of the field of computer vision in the 1960s. On the other hand, invariants were a very active mathematical subject in the latter half of the 19th century [6]. However, they were not derived through projections: they were derived on the assumption that 3-D information can be directly treated. Since we can actually treat only projected 2-D information, until recently only one geometric invariant [4], the cross ratio of four collinear points, was used in the field of computer vision. Only over the past few years have we highlighted other invariants.

During this time, several geometric invariants have been derived and are now being used in machine vision applications. For instance, two invariants of five coplanar points [1], two invariants of five coplanar lines [8], one invariant of two lines and two points on a plane [14], two invariants of two coplanar conics [5]. If we assume points, lines or conics in 3-D to be coplanar, there are such geometric invariants as listed above. This fact encouraged us to devote ourselves to finding geometric invariants for noncoplanar points, lines or conics. However, Burns–Wiess–Riseman [8] and Moses–Ullman [7] proved that one cannot calculate any invariant of the image of a set of general points in three dimensions from a single view; one requires at least two views. Then some assumptions were imposed on the distribution of noncoplanar points, lines, and conics to derive invariants. Rothwell–Forsyth–Zisserman–Mundy [9], [10] showed that there exist three geometric invariants of normal vectors of six planes for a trihedral object[1]. Sugimoto [11] showed that there exists one geometric invariant of five lines on two planes.

This paper provides a study on geometric invariants of noncoplanar lines on whose distribution some assumptions are imposed. As an extension of Sugimoto [11], it is shown that there exists one geometric invariant of six lines on three planes. The distribution of the six lines is completely clarified. Moreover, a condition for nonsingularity, i.e., well-definedness and nondegeneracy, of the invariant is also given. When we compared these results with those of Sugimoto [11], while the number of the lines increases, the assumptions imposed on the distribution of the lines are relaxed. Hence, the invariant derived in this paper can be applied to more general 3-D objects.

This paper is organized as follows: In Section 2 in preparation for further investigation, we describe a property of three lines on a plane. In Section 3 we first represent a line as the intersection of two planes and then consider a relationship between the parameters of planes and the vector that is obtained through observing the intersection line of the planes. Next we consider the change in the values of the parameters of planes over a number of viewpoints. In Section 4, we assume the values of the parameters of planes to be known and derive properties of the vectors that are obtained through observing the intersection lines. In Section 5, we eliminate the assumption; properties of the vectors are investigated without using the above assumption. And it is shown that there exists one geometric invariant of six lines on three

[1] If all the vertices of an object are characterized as the intersection of only three planes, it is called a trihedral object, or simply a trihedron.

planes. The distribution of the six lines is also clarified. Furthermore, the necessary and sufficient condition for nonsingularity of the invariant is given. Some experimental results with real images are presented in Section 6. In this paper, we assume that an object rigidly moves around a fixed viewpoint. We also assume that the focal length is the unit length and that the correspondence of lines among images is known.

2 Three lines on the image plane

Consider a perspective projection whose origin O coincides with the center of a lens and whose z axis is aligned with the optical axis (see Fig. 1). Then $z = 1$ is the image plane. Denote[2] by $\boldsymbol{X} = (X, Y, 1)^{\mathrm{T}}$ the coordinates of the image of a point (with coordinates $\boldsymbol{x} = (x, y, z)^{\mathrm{T}}$) in 3-D. Then we can easily see the following relationship:

$$X = \frac{x}{z}, \qquad Y = \frac{y}{z}. \tag{2.1}$$

When we observe a line $aX + bY + c = 0$ $(a^2 + b^2 \neq 0)$ on the image plane, we obtain a vector $(a, b, c)^{\mathrm{T}}$. Note that we can only determine it up to a scaling factor. The following fact is widely known for three different lines on the image plane.

Observation 2.1 Let three different lines i $(i = 1, 2, 3)$ on XY−plane be

$$a_i X + b_i Y + c_i \;=\; 0 \tag{2.2}$$

(where $a_i^2 + b_i^2 \neq 0$). Then they do not share a common point iff

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \;\neq\; 0. \tag{2.3}$$

□

Here we consider the meaning of the value of the left-hand side of (2.3). (2.2) of line i is rewritten as

$$(\, a_i,\ b_i,\ c_i)^{\mathrm{T}} \cdot \boldsymbol{X} \;=\; 0. \tag{2.4}$$

[2]We use a column vector.

Since $\boldsymbol{X}$ is the coordinates of any point on line i, $(a_i, b_i, c_i)^{\mathrm{T}}$ represents the normal vector of the plane on which both the origin and line i are. This plane is called the interpretation plane of line i (see Fig. 2). Therefore, for three lines i ($i = 1, 2, 3$), the value of the left-hand side of (2.3) represents the volume of a parallelepiped in three dimensions, which is constructed from the normal vectors of the interpretation planes of the three lines.

In this paper, we concentrate on this volume to derive an invariant.

Remark 2.1 As pointed out before, we can only determine vector $(a_i, b_i, c_i)^{\mathrm{T}}$ up to a scaling factor when we observe a line on the image plane. However, we can eliminate this indeterminacy by setting a criteria such as $a_i = 1$ or the normalization of the vector. □

3 Planes and lines in 3-D

3.1 Two planes and a line

A line in 3-D going through the origin or being on plane $z = 0$ makes just a point, or no image on the image plane. In this paper, a line is assumed neither to go through the origin nor to be on plane $z = 0$. In other words, a line is assumed to be perspectively projected to a line on the image plane. Such a line is called a line in a general position.

A line in a general position in 3-D is uniquely determined as a pair of planes, neither of which ever goes through the origin (see Fig. 2). Therefore, we represent a line as a pair of planes. Let two planes i ($i = 1, 2$) in 3-D be

$$a_i x + b_i y + c_i z + d_i \quad = \quad 0 \tag{3.1}$$

(where $d_i \cdot (a_i^2 + b_i^2 + c_i^2) \neq 0$). Denoting the normal vector of plane i by

$$\boldsymbol{n}_i \quad = \quad (a_i, b_i, c_i)^{\mathrm{T}} \quad (i = 1, 2),$$

we can rewrite (3.1) as

$$\boldsymbol{n}_i \cdot \boldsymbol{x} + d_i \quad = \quad 0. \tag{3.2}$$

Hence, $\boldsymbol{x}$, the coordinates of a point that is on both the planes, satisfies

$$\sum_{i=1}^{2} \lambda_i \left(\boldsymbol{n}_i \cdot \boldsymbol{x} + d_i \right) \quad = \quad 0, \tag{3.3}$$

where λ_i $(i = 1, 2)$ are real numbers. By fixing the values of λ_i $(i = 1, 2)$, we obtain the interpretation plane of the intersection line of planes 1 and 2:

$$(d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2) \cdot \boldsymbol{x} \;=\; 0. \tag{3.4}$$

Therefore, $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ is the normal vector of the interpretation plane of the intersection line of planes 1 and 2; we obtain $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ when we observe the line determined by $\boldsymbol{n}_i$ and $d_i\,(i = 1, 2)$.

Remark 3.1 Note that we have indeterminacy of a scaling factor between vector $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ and the vector we actually obtain as a result of observing the line. □

Remark 3.2 If we set $d_i = 0$ in (3.1), then all the lines on plane i are observed to be the same line on the image plane. This shows that the normal vectors of their interpretation planes coincide. □

3.2 Planes after a motion

Let a point (with coordinates $\boldsymbol{x}$) change its coordinates to $\boldsymbol{x}'$ after a motion. Here a rigid motion, i.e., a rotation around the viewpoint followed by a translation, is assumed to be admissible. Therefore,

$$\boldsymbol{x}' \;=\; R\boldsymbol{x} + \boldsymbol{t} \tag{3.5}$$

(where $R \in \mathrm{SO}(3)$, $\boldsymbol{t} \in \mathbf{R}^3$). Note that $\mathbf{R}$ and $\mathrm{SO}(3)$ denote the set of real numbers and the special orthogonal group of degree 3 over $\mathbf{R}$, respectively.

Plane i of (3.2) moves to

$$\boldsymbol{n}_i' \cdot \boldsymbol{x} + d_i' \;=\; 0 \tag{3.6}$$

after the motion of (3.5), where

$$\boldsymbol{n}_i' \;=\; R\boldsymbol{n}_i, \tag{3.7}$$

$$d_i' \;=\; d_i - (R^{\mathrm{T}}\boldsymbol{t}) \cdot \boldsymbol{n}_i. \tag{3.8}$$

Hence the relationship among the parameters that determine the plane before and after a motion is given by (3.7) and (3.8).

4 Properties of the plane parameters

In Section 3, we learned that when we observe the intersection line of two planes i, j (i, j are natural numbers), we obtain vector $\boldsymbol{n}_{ij}$, which is characterized by

$$\boldsymbol{n}_{ij} = d_j \boldsymbol{n}_i - d_i \boldsymbol{n}_j. \tag{4.1}$$

In addition, we also learned that plane i of (3.2) moves to that of (3.6) after a motion, and that $\boldsymbol{n}_i, \boldsymbol{n}'_i, d_i$ and d'_i satisfy (3.7) and (3.8). In this section, we investigate the properties of vectors $\boldsymbol{n}_{ij}$ when we are given certain planes in 3-D, i.e., $\boldsymbol{n}_i$ and d_i.

Suppose that four different planes $1, 2, 3, 4$ are given. We consider $\boldsymbol{n}_{12}, \boldsymbol{n}_{23}$ and $\boldsymbol{n}_{34}$, which are obtained by observing their intersection lines (see (4.1)), and then define a 3×3 matrix M_{1234} whose column vectors are these three vectors:

$$M_{1234} := [\, \boldsymbol{n}_{12} \mid \boldsymbol{n}_{23} \mid \boldsymbol{n}_{34} \,]. \tag{4.2}$$

Remark 4.1 When the three lines, i.e., the intersection line of planes 1 and 2, that of planes 2 and 3, and that of planes 3 and 4, satisfy one of the following:

(I) they share a common point in 3-D,

(II) they are parallel,

the value of the determinant of M_{1234} is zero since these three lines share a common point on the image plane through the perspective projection (see Observation 2.1). We assume that these three lines satisfy neither (I) nor (II). □

In a similar way, we consider $\boldsymbol{n}'_{12}, \boldsymbol{n}'_{23}$ and $\boldsymbol{n}'_{34}$ after a motion, and define a 3×3 matrix M'_{1234} characterized by them:

$$M'_{1234} := [\, \boldsymbol{n}'_{12} \mid \boldsymbol{n}'_{23} \mid \boldsymbol{n}'_{34} \,]. \tag{4.3}$$

M_{1234} and M'_{1234} have the following properties.

Lemma 4.1 (Sugimoto [11]) Let $\mathrm{rank} M_{1234} = 3$. Then,

$$\mathrm{rank} M'_{1234} = 3. \tag{4.4}$$

$$d_2\, d_3\, \det M'_{1234} = d'_2\, d'_3\, \det M_{1234}. \tag{4.5}$$

□

Defining M_{1235} and M'_{1235}, we have the following lemma (see Remark 4.2).

Lemma 4.2 (Sugimoto [11]) For $M_{1234}, M_{1235}, M'_{1234}$ and M'_{1235},

$$\det M'_{1234} \det M_{1235} = \det M_{1234} \det M'_{1235}. \tag{4.6}$$

□

When four different planes are planes $7, 6, 3, 4$, we similarly obtain the following equation as a counterpart of (4.6):

$$\det M'_{7634} \det M_{7635} = \det M_{7634} \det M'_{7635}. \tag{4.7}$$

Remark 4.2 If $d'_2 = 0$, then both the intersection line of planes 1 and 2, and that of planes 2 and 3, are observed to be coincident after a motion (see Remark 3.2). On the other hand, if $d'_3 = 0$, then both the intersection line of planes 2 and 3, and that of planes 3 and 4, are observed to be coincident. These facts show that if $d'_2 \cdot d'_3 = 0$ then the number of visible lines changes before and after a motion. In this paper, we do not assume such a change occurs, which leads to $d'_2 \cdot d'_3 \neq 0$. Similarly $d'_6 \neq 0$. □

5 Invariant of lines on three planes

Here, we investigate the properties of the vectors we actually obtain, without assuming the known values of $\boldsymbol{n}_i$ and d_i.

5.1 Invariant of six lines

As stressed before, there is a scaling indeterminacy between vector $\boldsymbol{n}_{ij}$ that is derived from planes i, j (i, j are natural numbers), and the vector we actually obtain through observing their intersection line (see Remark 3.1). Hence, denote by $\boldsymbol{N}_{ij}$ the vector we actually obtain through observing the intersection line. Then

$$\boldsymbol{N}_{ij} = \rho_{ij} \boldsymbol{n}_{ij} \quad (\rho_{ij} \neq 0) \tag{5.1}$$

is satisfied. Here ρ_{ij} is a scaling factor and its value is not known. Define N_{ijkl} (i, j, k, l are natural numbers) as a counterpart of M_{ijkl}:

$$N_{ijkl} := [\, \boldsymbol{N}_{ij} \mid \boldsymbol{N}_{jk} \mid \boldsymbol{N}_{kl} \,]. \tag{5.2}$$

(5.1) and (5.2) yield

$$\det N_{ijkl} \quad = \quad \rho_{ij} \cdot \rho_{jk} \cdot \rho_{kl} \cdot \det M_{ijkl}. \tag{5.3}$$

We denote by $\boldsymbol{N}'_{ij}$ the vector we actually obtain after a motion, and similarly define N'_{ijkl}. Note that $\boldsymbol{N}'_{ij} = \rho'_{ij}\boldsymbol{n}'_{ij}$ ($\rho'_{ij} \neq 0$), where the value of ρ'_{ij} is unknown. N_{ijkl} and N'_{ijkl} have the following properties.

Theorem 5.1 Let $\mathrm{rank}N_{ij3k} = 3$ ($i = 1,7$; $j = 2,6$; $k = 4,5$). Then,

$$\mathrm{rank}N'_{ij3k} \quad = \quad 3, \tag{5.4}$$

$$\frac{\det N_{1234} \cdot \det N_{7635}}{\det N_{1235} \cdot \det N_{7634}} \quad = \quad \frac{\det N'_{1234} \cdot \det N'_{7635}}{\det N'_{1235} \cdot \det N'_{7634}}. \tag{5.5}$$

Proof: Definition of N_{ijkl} leads to $\mathrm{rank}N_{ijkl} = \mathrm{rank}M_{ijkl}$. Similarly, $\mathrm{rank}N'_{ijkl} = \mathrm{rank}M'_{ijkl}$. These yield (5.4) from Lemma 4.1.

From (5.3) we obtain[3]

$$\text{LHS of (5.5)} \quad = \quad \frac{\det M_{1234} \cdot \det M_{7635}}{\det M_{1235} \cdot \det M_{7634}}.$$

On the other hand, it is easy to see

$$\text{RHS of (5.5)} \quad = \quad \frac{\det M'_{1234} \cdot \det M'_{7635}}{\det M'_{1235} \cdot \det M'_{7634}}.$$

These yield (5.5) from (4.6) and (4.7). □

Theorem 5.1 shows that there exists a geometric invariant

$$I \quad := \quad \frac{\det N_{1234} \cdot \det N_{7635}}{\det N_{1235} \cdot \det N_{7634}} \tag{5.6}$$

for six lines, which are the intersections of proper pairs of seven planes. It is easy to see that the value of invariant I generally depends on a choice of six lines there. This indicates that an object generically has its own value of invariant I.

In the subsequent sections, we clarify the distribution of the above six lines and give a necessary and sufficient condition under which the invariant is nonsingular.

[3]LHS and RHS stand for the left-hand side and the right-hand side, respectively.

5.2 Distribution of six lines

Geometric invariant I is calculated from the following six lines:

- L_{12} (the intersection line of planes 1 and 2),
- L_{23} (the intersection line of planes 2 and 3),
- L_{34} (the intersection line of planes 3 and 4),
- L_{35} (the intersection line of planes 3 and 5),
- L_{76} (the intersection line of planes 7 and 6),
- L_{63} (the intersection line of planes 6 and 3).

The distribution of these six lines in 3-D is characterized as follows:

1. The six lines are all on plane 2, plane 3 or plane 6,
2. The intersection line of planes 2 and 3 is included (L_{23}),
3. The intersection line of planes 6 and 3 is included (L_{63}),
4. There are two lines, L_{34} and L_{35}, on plane 3 in addition to L_{23} and L_{63},
5. There is one line, L_{12}, on plane 2 in addition to L_{23},
6. There is one line, L_{76}, on plane 6 in addition to L_{63}.

Therefore, there exists a geometric invariant I for six lines on three planes (planes 2, 3 and 6 above). The six lines include, for the three aligned planes, 1) the two intersection lines of the adjacent planes in the alignment; 2) two other lines on the middle plane; and 3) one other line on each side plane (see Fig. 3).

5.3 Condition for nonsingularity

In this subsection, we give the necessary and sufficient condition under which the invariant I is nonsingular: the condition for nonsingularity of I. Here we define "An invariant is nonsingular" as "The value of the invariant is not 0, ∞ or $^0/_0$". From (5.6) it is easy to see that

I is nonsingular iff the value of the determinant of N_{ijkl} is not zero. Therefore, the necessary and sufficient condition under which I is nonsingular is that the value of the determinant of M_{ijkl} is not zero (see (5.3)).

Observation 2.1 tells us that when three lines on the image plane do not share a common point, the value of the determinant of M_{ijkl} is never zero, which yields the following theorem (see Remark 4.1).

Theorem 5.2 Let A be the plane where four lines among the six are, and B, C be the other planes. The necessary and sufficient condition under which I in (5.6) is nonsingular is that the six lines on three planes A, B and C have the following property:

For three lines, i.e., the two lines of plane B [C] and any of the lines on plane A that is neither the intersection of A and B, nor that of A and C, (I) and (II) are satisfied.

(I) They never share a common point in 3-D.

(II) They are not parallel. □

Remark 5.1 When we observe seven points on three aligned planes such that

- there are two points on each intersection line of the two adjacent planes in the alignment and,
- there is one other point on each plane,

it is easy to see that we can construct six lines on three planes, from which we can calculate invariant I and which satisfy the condition for nonsingularity (see Fig. 4). This shows that there exists the same invariant for seven points on three planes. □

6 Experimental results

In Section 5, we proved the existence of geometric invariant I of six lines on three planes. When the three planes are aligned, the six lines there include 1) the two intersection lines of the adjacent planes in the alignment; 2) two other lines on the middle plane; and 3) one other line on each side plane. In addition, to guarantee nonsingularity of the invariant, two conditions have to be satisfied: for three lines, i.e., the two coplanar lines on either of the side planes and any of the lines on the middle plane, which is not an intersection line, 4) they never share a common point in 3-D; and 5) they are not parallel. On the basis of these results, our experimental results with real images are shown.

Two objects, polygons I and II, that are used to calculate values of the invariant are shown in Figs. 5 and 6. These two objects are constructed from a parallelepiped or a rectangular parallelepiped on which the same triangular prism is attached. We can assume that they are similar to each other.

We obtained several images[4] of polygon I and II using a fixed camera. These polygons were randomly moved by hand. For each image, we first applied a low pass filter of a 3×3 weighted kernel window to reduce noise. We then calculated the Laplacian with an 8-neighbor weighted coefficient matrix to extract the edges (Figs. 8, 9). Next, to each edge in the image, we applied the method of least squares to find the equation of the line representing the edge. On the other hand, we attached labels to the planes and edges of the polygons (see Fig. 7) and chose planes (A), (B) and (C) as the three planes. We then selected six of ten lines[5] $1, 2, \ldots, 10$, which satisfy the condition for nonsingularity of the invariant, on either plane (A), (B) or (C) to calculate the value of invariant I. There are three combinations[6] of six of the ten lines that include line 4, the intersection of (A) and (B), line 6, the intersection of (A) and (C), two other lines on (A) and one other line on each of (B) and (C). Thus, for the lines that were obtained from six edge images (a), ..., (f) in Fig. 8, we calculated the values of these three invariants, which are shown in Table 1. We denote by I_{ijklmn} the invariant of six lines i, j, k, l, m, n ($i, j, k, l, m, n \in \{1, 2, \ldots, 10\}$). For each invariant, we also showed the mean m over the six images, the standard deviation σ and the percentage of the standard deviation to the mean. Similarly the values for the six edge images in Fig. 9 are shown in Table 2.

Tables 1 and 2 show that essentially all of the values of I_{ijklmn} are constants: they remain stable in spite of a change in viewpoint. This shows that the values of the invariant are reliable even for noisy images. Furthermore, their values significantly depend on the object even though the two objects are similar to each other; for each object, they also depend on

[4]Each image consists of 480×512 pixels. And each pixel is assigned a natural number from $0 \sim 255$ as the value of its grey level.

[5]The line i denotes the line representing the edge i ($i \in \{1, 2, \ldots, 10\}$).

[6]Since three lines 5,6,8 and three lines 6,7,10 respectively share common points in 3-D, a combination where these three lines are included does not satisfy the condition for nonsingularity.

a choice of six lines, i.e., a combination of the observed lines. These show that each object generically has its own value of I. Therefore, the value of I can be important in identifying one object out of many.

As shown above, for a real 3-D object we found the invariant I that is unaffected by a change in viewpoint and has its own value.

7 Conclusion

We proved the existence of one geometric invariant I of six lines on three planes. When the three planes are aligned, these six lines include 1) the two intersection lines of the adjacent planes in the alignment; 2) two other lines on the middle plane; and 3) one other line on each side plane. Furthermore, the necessary and sufficient conditions for nonsingularity of the invariant are for three lines, i.e., the two coplanar lines on either of the side planes and any of the lines on the middle plane, which is not an intersection line, 4) they never share a common point in 3-D; and 5) they are not parallel.

We applied these theoretical results to real images, and found that the values of the invariant remain stable even for noisy images. Furthermore, we also found that an object generically has its own value of the invariant. Left for future investigation is the theoretical analysis of the noise sensitivity of the invariant.

When we have no assumption on a distribution of points in 3-D, we can derive no promising geometric invariant from a single view [7], [8]. However, if there exist functions, so called quasi-invariants [2], which are unaffected by most changes in the viewpoint, they would be definitely very useful even though we cannot expect the same value over all changes in the viewpoint. We are now going to search for quasi-invariants and to characterize the region of the viewpoint where the value of the quasi-invariant never changes.

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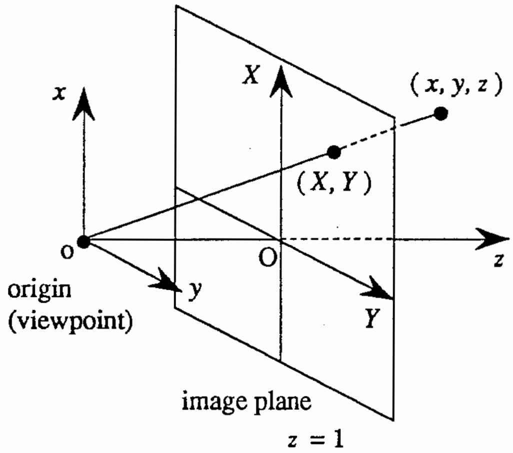

Fig. 1: Perspective projection centered at the origin

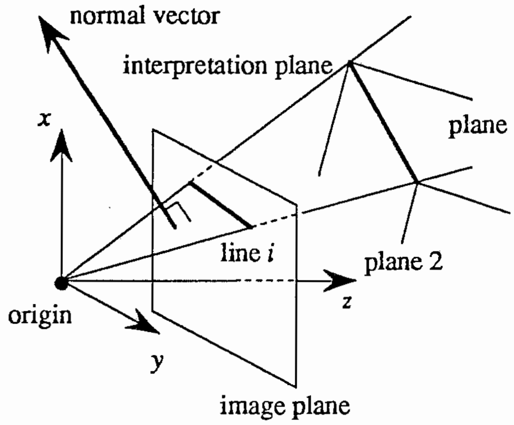

Fig. 2: Line i and the normal vector of its interpretation plane

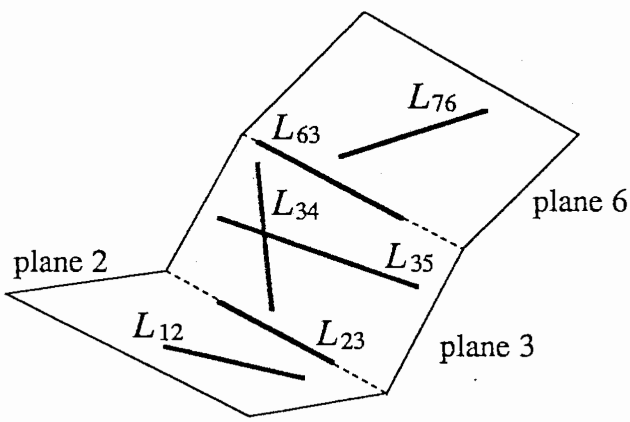

Fig. 3: Six lines on three planes

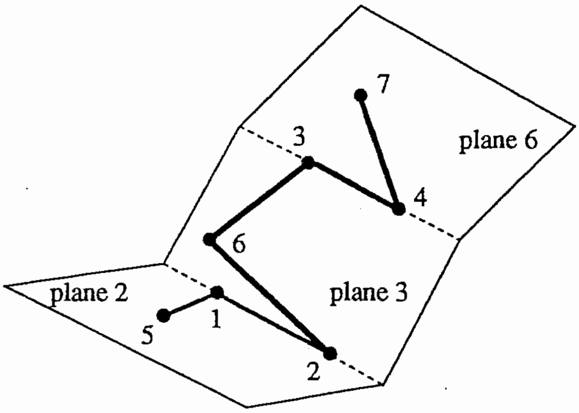

Fig. 4: Six lines derived from seven points on three planes

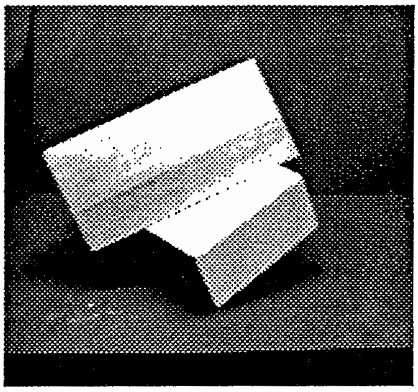

Fig. 5: A polygon I used to calculate geometric invariant

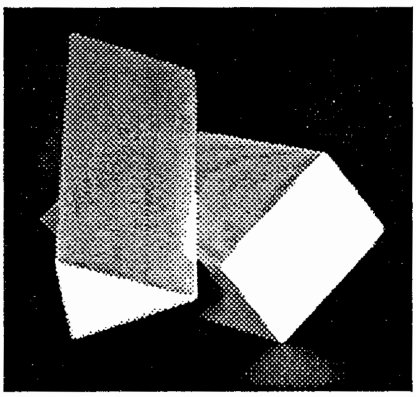

Fig. 6: A polygon II used to calculate geometric invariant

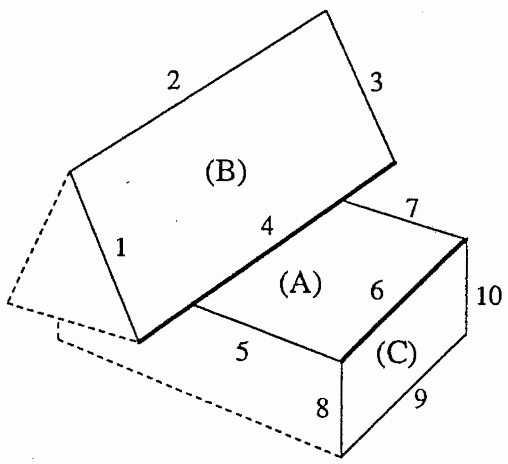

Fig. 7: Labels for planes and lines of the polygon in Figs. 4 and 5

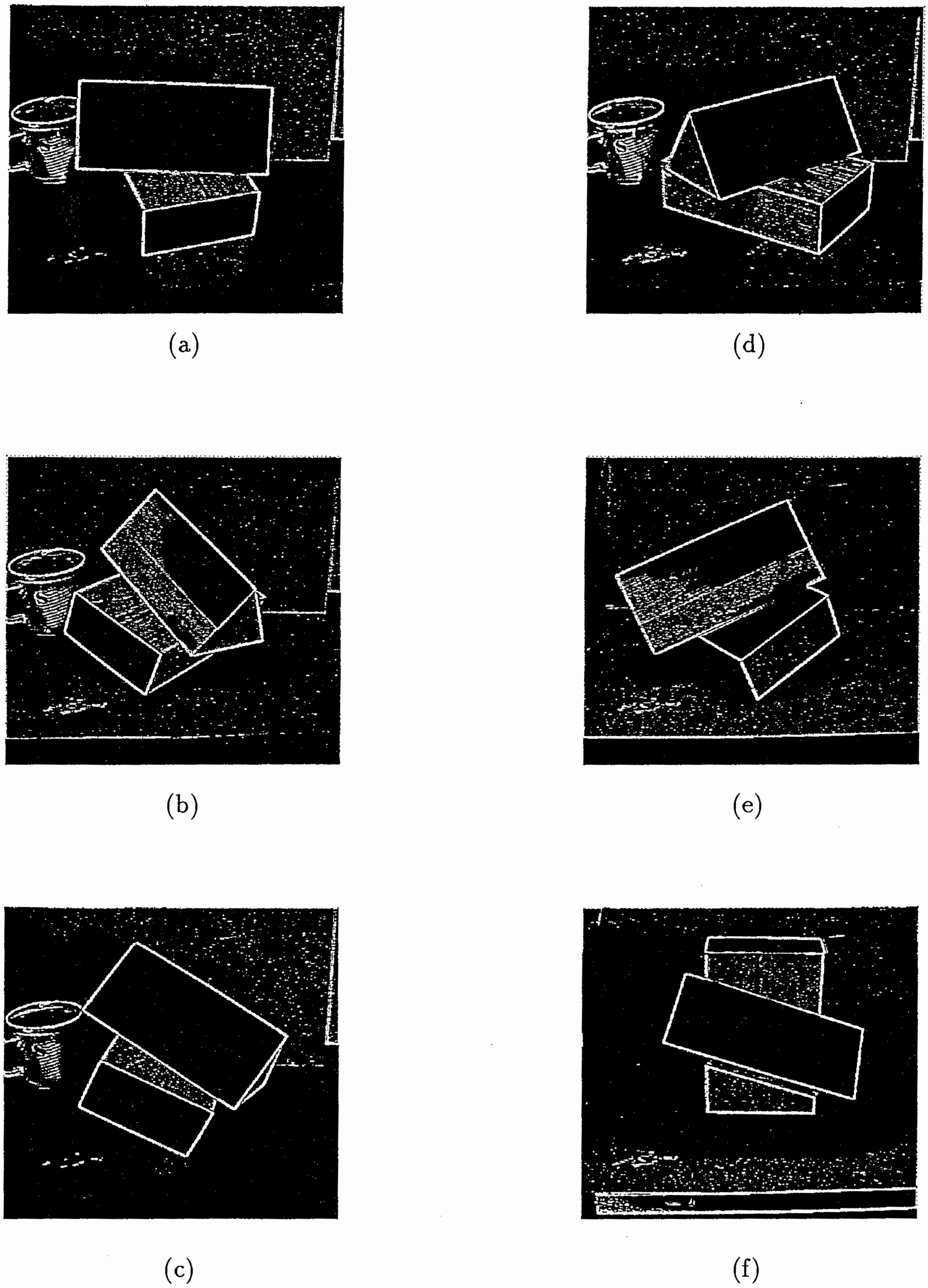

Fig. 8: Extracted edges from images of the polygon I

We obtained several images of the polygon in Fig. 5 using a fixed camera. The polygon was moved randomly by hand. For each image, we first applied a low pass filter of a 3×3 weighted kernel window to reduce noise. We then calculated the Laplacian with an 8-neighbor weighted coefficient matrix to extract edges ((e) is the extracted edge image for Fig. 5).

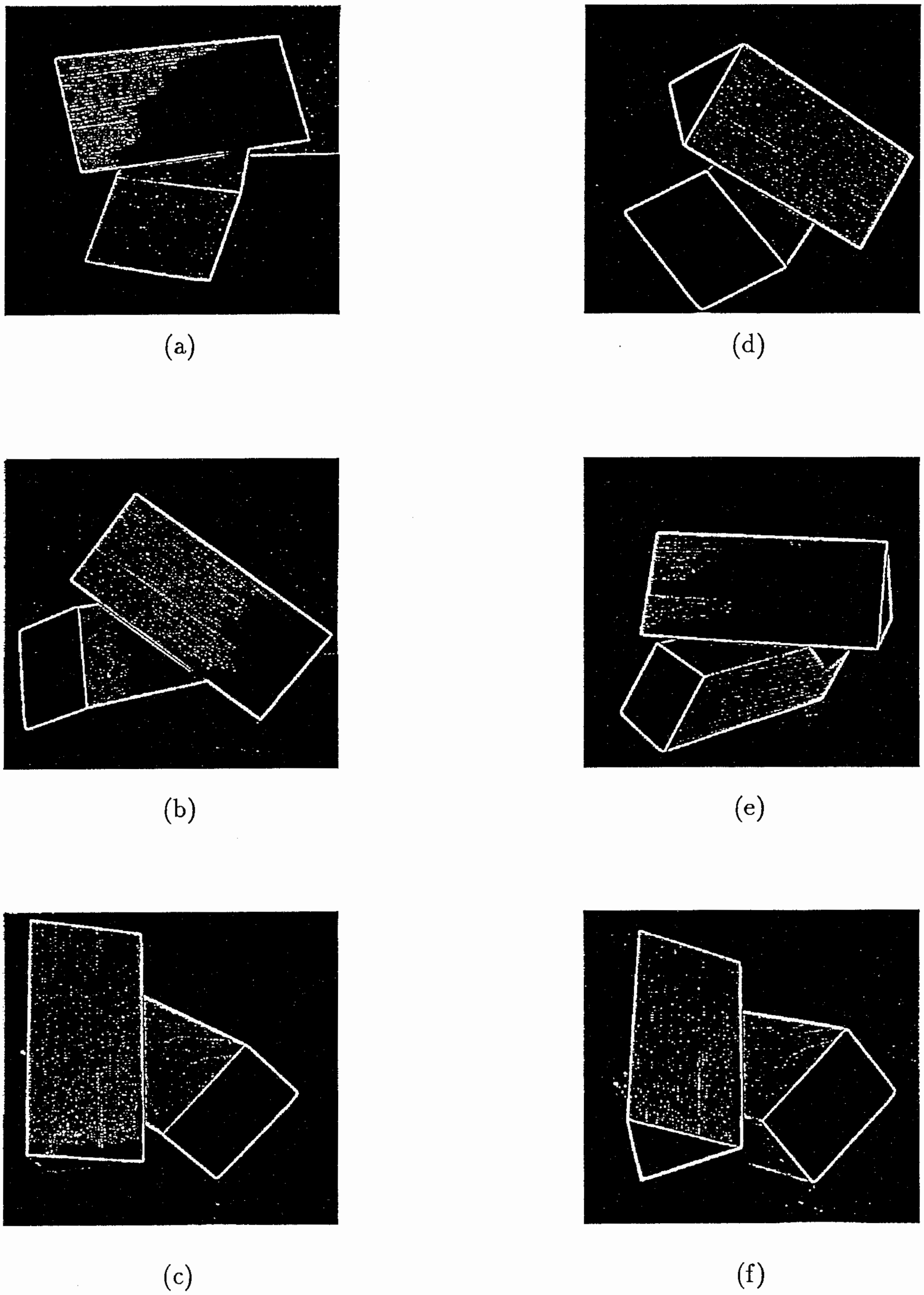

Fig. 9: Extracted edges from images of the polygon II

We obtained several images of the polygon in Fig. 6 using a fixed camera. The polygon was moved randomly by hand. For each image, we first applied a low pass filter of a 3×3 weighted kernel window to reduce noise. We then calculated the Laplacian with an 8-neighbor weighted coefficient matrix to extract edges ((f) is the extracted edge image for Fig. 6).

Table 1: Values of invariant I for the polygon I

Shown are the values of I calculated from the lines representing the edges in Fig. 8. Also shown are their means m, standard deviations σ and the percentages of the standard deviations of the means.

	I_{145679}	I_{245679}	I_{345679}
(a)	0.283451	0.993436	6.523431
(b)	0.275090	0.974098	6.936328
(c)	0.287415	1.006754	6.726357
(d)	0.258875	0.927373	5.959299
(e)	0.269928	0.984172	6.787387
(f)	0.289249	1.026566	6.982462
m	0.277335	0.985400	6.652544
σ	0.010659	0.030852	0.344063
σ/m (%)	3.84	3.13	5.17

Table 2: Values of invariant I for the polygon II

Shown are the values of I calculated from the lines representing the edges in Fig. 9. Also shown are their means m, standard deviations σ and the percentages of the standard deviations of the means.

	I_{145679}	I_{245679}	I_{345679}
(a)	0.226883	0.998656	2.942333
(b)	0.229346	0.973880	2.929213
(c)	0.224297	0.979282	2.874939
(d)	0.229489	1.024318	3.044252
(e)	0.244337	0.991458	3.080928
(f)	0.221639	1.022973	2.996958
m	0.229332	0.998428	2.978104
σ	0.007254	0.019535	0.070260
σ/m (%)	3.16	1.96	2.36