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Projective Invariant of Lines on Adjacent Planar Regions in a Single View

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1993.10.12

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Abstract

The importance of projective invariants to many machine vision tasks such as model-based recognition has been recognized. A number of recent studies on projective invariants in a single view concentrate on coplanar objects: coplanar points, coplanar lines, coplanar points and lines, coplanar conics, etc. This paper presents a study on projective invariants of noncoplanar objects, i.e., 3-D objects. A new projective invariant is derived from five lines on two adjacent planar regions in a single view. The condition under which the invariant is nonsingular is also described. In addition, we present some experimental results with real images and find that the values of the invariant over a number of viewpoints remain stable even for noisy images and that a 3-D object has its own proper value of the invariant. Therefore, we no longer need assume coplanar objects. We can directly treat 3-D objects to calculate projective invariants.

Key Words: projective invariant, adjacent planar regions, nonsingularity, 3-D object recognition.

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1 Introduction

We human beings can easily recognize objects in 3-D through visual 2-D information. Information projected onto the image plane depends on the position of an object relative to a viewpoint, which results in numerous different images, even for the same object. This complicates many problems in machine vision. Therefore, extracting the properties that remain invariant under any change in viewpoint would provide useful information. In other words, it leads to powerful methods [6], [10] of supporting a number of machine vision tasks such as object recognition to extract projective invariants, i.e., functions that are unaffected by a change in viewpoint and which are characterized by images of points, lines or curves constructing an object. For instance, a typical approach to model-based object recognition is divided into two procedures: for a given image, 1) to determine the position of an object relative to a viewpoint, i.e., *pose determination*; and then 2) to compare the given image of an object with one that is stored in a library of images to identify the object. However, if we can calculate the projective invariants of an object, attaching the values of the invariants to images in the library allows a reduction in the number of images to be compared without executing procedure 1), and makes it possible to directly compare the given image with one in the library [2], [9]. As for the problem of model description, how to describe the shape of an object is the main concern. Using invariant shape descriptors is definitely efficient since such descriptions are unaffected by a change in viewpoint. As has been seen, projective invariants are not only important but are also readily applicable to the field of computer vision.

From this point of view, the importance of invariants has been recognized since the origin of the field of computer vision in the 1960s. On the other hand, projective invariants were a very active mathematical subject in the latter half of the 19th century. However, until recently only one projective invariant [3], the cross ratio of four points on a line, had been used in the field of computer vision. Only over the past few years have we highlighted other invariants.

During this time, several projective invariants have been derived and are now being used in machine vision applications. For instance, two invariants of five points on a plane [1], two invariants of five lines on a plane [6], one invariant of two lines and two points on a plane [11], two invariants of two coplanar conics [4]. If we assume an object to be coplanar, there are many such projective invariants, as listed above. This fact encouraged us to devote ourselves to finding projective invariants for 3-D objects. However, Burns–Wiess–Riseman [6] and Moses–Ullman [5] proved that one cannot calculate any invariant of the image of a set of general points in three dimensions from a single view; one requires at least two views. However, this does not necessarily indicate that one cannot calculate any invariant for a 3-D object from a single view even if we impose an assumption on an object. In fact, Rothwell–Forsyth–Zisserman–Mundy [7], [8] showed that there exist three projective invariants of normal vectors of six planes for a trihedral object[1].

This paper is a study on projective invariants of 3-D objects on which some assumptions are imposed. It is shown that there exists one projective invariant of five lines on two adjacent planar regions. Moreover, a condition for nonsingularity, i.e., well-definedness and nondegeneracy, of the invariant is also given. Since the set of polygons with two adjacent planar regions (five lines are assumed to exist) includes the set of trihedrons (six planes are assumed to exist), the invariant derived in this paper can be applied to more general 3-D objects.

This paper is organized as follows: In Section 2 in preparation for further investigation, we describe a property of three lines on a plane. In Section 3 we first regard a line as the intersection of two planes and then consider a relationship between the parameters that determine the planes and the vector that is obtained through observing the intersection line of the planes. Next we consider the change in the values of the parameters of planes over a number of viewpoints. In Section 4, we assume the values of the parameters of planes to be known and derive properties that are characterized by the vectors obtained through observing the intersection lines. In Section 5, we eliminate the assumption; properties of the vectors are investigated without using the above assumption. And it is shown that there exists one projective invariant of five lines on two adjacent planar regions. Furthermore, a necessary and sufficient condition for nonsingularity of the invariant is also given. Some experimental results with real images are presented in Section 6. In this paper, we assume that an object rigidly moves around a fixed viewpoint. We also assume that the focal length is the unit length and that the correspondence of lines among images is known.

[1]If all the vertices of an object are characterized as the intersection of only three planes, it is called a trihedral object, or simply a trihedron.

2 Three lines on the image plane

Consider a perspective projection whose origin O coincides with the center of a lens and whose z axis is aligned with the optical axis (see Fig. 1). Then $z = 1$ is the image plane. Denote[2] by $\boldsymbol{X} = (X, Y, 1)^{\mathrm{T}}$ the coordinates of the image of a point (with coordinates $\boldsymbol{x} = (x, y, z)^{\mathrm{T}}$) in 3-D. Then we can easily see the following relationship:

$$X = \frac{x}{z}, \qquad Y = \frac{y}{z}. \tag{2.1}$$

When we observe a line $aX + bY + c = 0$ $(a^2 + b^2 \neq 0)$ on the image plane, we obtain a vector $(a, b, c)^{\mathrm{T}}$. Note that we can only determine it up to a scaling factor. The following fact is widely known for three different lines on the image plane.

Observation 2.1 Let three different lines i $(i = 1, 2, 3)$ on $XY-$plane be

$$a_i X + b_i Y + c_i \;=\; 0 \tag{2.2}$$

(where $a_i^2 + b_i^2 \neq 0$). Then the necessary and sufficient condition under which they do not share a common point is

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \;\neq\; 0. \tag{2.3}$$

□

Here we consider the meaning of the value of the left-hand side of (2.3). (2.2) of line i is rewritten as

$$(\, a_i,\, b_i,\, c_i)^{\mathrm{T}} \cdot \boldsymbol{X} \;=\; 0. \tag{2.4}$$

Since $\boldsymbol{X}$ is the coordinates of any point on line i, $(a_i, b_i, c_i)^{\mathrm{T}}$ represents the normal vector of the plane on which both the origin and line i are. This plane is called the interpretation plane of line i (see Fig. 2). Therefore, for three lines i $(i = 1, 2, 3)$, the value of the left-hand side of (2.3) represents the volume of a parallelepiped in three dimensions, which is constructed by the normal vectors of the interpretation planes of the lines.

In this paper, we concentrate on this volume to derive an invariant.

---

[2]We use a column vector.

Remark 2.1 As pointed out before, we can only determine vector $(a_i, b_i, c_i)^{\mathrm{T}}$ up to a scaling factor when we observe a line on the image plane. However, we can eliminate this indeterminacy by setting a criteria such as $a_i = 1$ or the normalization of the vector. □

3 Planes and lines in 3-D

3.1 Line as the intersection of two planes

A line in 3-D going through the origin or being on plane $z = 0$ makes just a point, or no image on the image plane. In this paper, a line is assumed neither to go through the origin nor to be on plane $z = 0$. In other words, a line is assumed to be perspectively projected to a line on the image plane. Such a line is called a line in a general position.

A line in a general position in 3-D is uniquely determined as a pair of planes, each of which never goes through the origin (see Fig. 2). Therefore, we represent a line as a pair of planes. Let two planes i ($i = 1, 2$) in 3-D be

$$a_i x + b_i y + c_i z + d_i \quad = \quad 0 \tag{3.1}$$

(where $d_i \cdot (a_i^2 + b_i^2 + c_i^2) \neq 0$). Denote by $\boldsymbol{n}_i$ the normal vector of plane i, then

$$\boldsymbol{n}_i \quad = \quad (a_i, b_i, c_i)^{\mathrm{T}}.$$

And (3.1) is rewritten as

$$\boldsymbol{n}_i \cdot \boldsymbol{x} + d_i \quad = \quad 0. \tag{3.2}$$

Hence, $\boldsymbol{x}$, the coordinates of a point which is on both the planes, satisfies

$$\sum_{i=1}^{2} \lambda_i \left(\boldsymbol{n}_i \cdot \boldsymbol{x} + d_i \right) \quad = \quad 0, \tag{3.3}$$

where λ_i ($i = 1, 2$) are real numbers.

As seen above, the vector that is obtained when a line is observed on the image plane is the normal vector of the interpretation plane of the line. By fixing the values of λ_i ($i = 1, 2$) so that the coordinates of the origin O satisfy (3.3), we obtain the interpretation plane of the intersection line of planes 1 and 2:

$$(d_2 \boldsymbol{n}_1 - d_1 \boldsymbol{n}_2) \cdot \boldsymbol{x} \quad = \quad 0. \tag{3.4}$$

Therefore, $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ is the normal vector of the interpretation plane of the intersection line of planes 1 and 2; we obtain $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ when we observe the line determined by $\boldsymbol{n}_i$ and $d_i\,(i=1,2)$.

Remark 3.1 Note that we have indeterminacy of a scaling factor between vector $d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2$ and the vector we actually obtain as a result of observing the line. □

Remark 3.2 If we set $d_i = 0$ in (3.1), then all the lines on plane i are observed to be the same line on the image plane. This shows that the normal vectors of their interpretation planes coincide. □

3.2 Planes after a motion

Let a point (with coordinates $\boldsymbol{x}$) change its coordinates to $\boldsymbol{x}'$ after a motion. Here a rigid motion, i.e., a rotation around the viewpoint followed by a translation, is assumed to be admissible. Therefore,

$$\boldsymbol{x}' = R\boldsymbol{x} + \boldsymbol{t} \tag{3.5}$$

(where $R \in \mathrm{SO}(3),\ \boldsymbol{t} \in \mathbf{R}^3$). Note that $\mathbf{R}$ and SO(3) denote the set of real numbers and the special orthogonal group of degree 3 over $\mathbf{R}$, respectively.

Substituting (3.5) into (3.2), it follows that

$$\begin{aligned}(3.2) \iff\ & \boldsymbol{n}_i \cdot R^{-1}(\boldsymbol{x}' - \boldsymbol{t}) + d_i = 0 \\ \iff\ & (R\boldsymbol{n}_i)\cdot(\boldsymbol{x}' - \boldsymbol{t}) + d_i = 0 \\ \iff\ & (R\boldsymbol{n}_i)\cdot \boldsymbol{x}' + \left\{d_i - (R^{\mathrm{T}}\boldsymbol{t})\cdot n_i\right\} = 0.\end{aligned}$$

Therefore, plane i of (3.2) moves to

$$\boldsymbol{n}_i' \cdot \boldsymbol{x} + d_i' = 0 \tag{3.6}$$

after the motion of (3.5), where

$$\boldsymbol{n}_i' = R\boldsymbol{n}_i, \tag{3.7}$$

$$d_i' = d_i - (R^{\mathrm{T}}\boldsymbol{t})\cdot n_i. \tag{3.8}$$

Hence the relationship among the parameters that determine the plane before and after a motion is given by (3.7) and (3.8).

4 Properties on the image plane derived from planes

In Section 3, we learned that when we observe the intersection line of two planes i, j (i, j are natural numbers), we obtain vector $\boldsymbol{n}_{ij}$, which is characterized by

$$\boldsymbol{n}_{ij} = d_j \boldsymbol{n}_i - d_i \boldsymbol{n}_j. \tag{4.1}$$

In addition, we also learned that plane i of (3.2) moves to that of (3.6) after a motion, and that $\boldsymbol{n}_i, \boldsymbol{n}'_i, d_i, d'_i$ satisfy (3.7) and (3.8). In this section, we investigate the properties of vectors $\boldsymbol{n}_{ij}$ when we are given certain planes in 3-D, i.e., $\boldsymbol{n}_i$ and d_i.

Suppose that four different planes $1, 2, 3, 4$ are given. We consider $\boldsymbol{n}_{12}, \boldsymbol{n}_{23}$ and $\boldsymbol{n}_{34}$, which are obtained by observing their intersection lines (see (4.1)), and then define a 3×3 matrix M_{1234} whose column vectors are these three vectors:

$$M_{1234} := [\, \boldsymbol{n}_{12} \mid \boldsymbol{n}_{23} \mid \boldsymbol{n}_{34} \,]. \tag{4.2}$$

Remark 4.1 When the three lines, i.e., the intersection line of planes 1 and 2, that of planes 2 and 3, and that of planes 3 and 4, satisfy one of the following:

(I) they share a common point in 3-D,

(II) they are parallel,

the value of the determinant of M_{1234} is zero since these three lines share a common point on the image plane through the perspective projection (see Observation 2.1). We assume that these three lines satisfy neither (I) nor (II). □

In a similar way, we consider $\boldsymbol{n}'_{12}, \boldsymbol{n}'_{23}$ and $\boldsymbol{n}'_{34}$, and define a 3×3 matrix M'_{1234} characterized by them:

$$M'_{1234} := [\, \boldsymbol{n}'_{12} \mid \boldsymbol{n}'_{23} \mid \boldsymbol{n}'_{34} \,]. \tag{4.3}$$

M_{1234} and M'_{1234} have the following properties.

Lemma 4.1 Let $\mathrm{rank} M_{1234} = 3$. Then,

$$\mathrm{rank} M'_{1234} = 3. \tag{4.4}$$

$$d_2\, d_3 \det M'_{1234} = d'_2\, d'_3 \det M_{1234}. \tag{4.5}$$

Proof: Since the properties (I) and (II) in Remark 4.1 for three lines remain invariant after any motion, we immediately obtain (4.4) (see Observation 2.1).

Let $D_{ijk} := \det[\, \boldsymbol{n}_i \mid \boldsymbol{n}_j \mid \boldsymbol{n}_k \,]$ ($i,j,k \in \{1,2,3,4\}$). Then we obtain

$$\begin{aligned}
\det M_{1234} &= \det[\, d_2\boldsymbol{n}_1 - d_1\boldsymbol{n}_2 \mid d_3\boldsymbol{n}_2 - d_2\boldsymbol{n}_3 \mid d_4\boldsymbol{n}_3 - d_3\boldsymbol{n}_4 \,] \\
&= d_2d_3d_4D_{123} + d_2d_3(-d_3)D_{124} + d_2(-d_2)(-d_3)D_{134} + (-d_1)(-d_2)(-d_3)D_{234} \\
&= d_2d_3\,\{d_4D_{123} - d_3D_{124} + d_2D_{134} - d_1D_{234}\}\,. \qquad (4.6)
\end{aligned}$$

Similarly, for $i,j,k \in \{1,2,3,4\}$ define $D'_{ijk} := \det[\, \boldsymbol{n}'_i \mid \boldsymbol{n}'_j \mid \boldsymbol{n}'_k \,]$. Since (3.7) yields $D'_{ijk} = D_{ijk}$,

$$\begin{aligned}
\det M'_{1234} &= d'_2d'_3\,\{d'_4D'_{123} - d'_3D'_{124} + d'_2D'_{134} - d'_1D'_{234}\} \\
&= d'_2d'_3\,[\{d_4D_{123} - d_3D_{124} + d_2D_{134} - d_1D_{234}\} - \Delta]\,, \qquad (4.7)
\end{aligned}$$

where

$$\Delta = \left\{(R^{\mathrm{T}}\boldsymbol{t})\cdot\boldsymbol{n}_4\right\}D_{123} - \left\{(R^{\mathrm{T}}\boldsymbol{t})\cdot\boldsymbol{n}_3\right\}D_{124} + \left\{(R^{\mathrm{T}}\boldsymbol{t})\cdot\boldsymbol{n}_2\right\}D_{134} - \left\{(R^{\mathrm{T}}\boldsymbol{t})\cdot\boldsymbol{n}_1\right\}D_{234}.$$

It is easy to see

$$\begin{aligned}
\Delta &= \det[\, \boldsymbol{n}_1 \mid \boldsymbol{n}_2 \mid (R^{\mathrm{T}}\boldsymbol{t}) \times (\boldsymbol{n}_3 \times \boldsymbol{n}_4) \,] + \det[\, (R^{\mathrm{T}}\boldsymbol{t}) \times (\boldsymbol{n}_1 \times \boldsymbol{n}_2) \mid \boldsymbol{n}_3 \mid \boldsymbol{n}_4 \,] \\
&= (\boldsymbol{n}_1 \times \boldsymbol{n}_2)\cdot(R^{\mathrm{T}}\boldsymbol{t}) \times (\boldsymbol{n}_3 \times \boldsymbol{n}_4) + (R^{\mathrm{T}}\boldsymbol{t}) \times (\boldsymbol{n}_1 \times \boldsymbol{n}_2)\cdot(\boldsymbol{n}_3 \times \boldsymbol{n}_4) \\
&= 0,
\end{aligned}$$

which yields (4.5) from (4.6) and (4.7). □

Lemma 4.1 shows that the ratio of the value of the determinant of M_{1234} to that of M'_{1234} is dependent only on planes 2 and 3; it is independent of the choice of planes 1 and 4. Hence, we replace plane 4 with another plane, plane 5, which yields

$$d_2\ d_3\ \det M'_{1235} = d'_2\ d'_3\ \det M_{1235}. \qquad (4.8)$$

This is combined with (4.5) to obtain the following lemma (see Remark 4.2).

Lemma 4.2 For $M_{1234}, M_{1235}, M'_{1234}$ and M'_{1235},

$$\det M'_{1234}\ \det M_{1235} = \det M_{1234}\ \det M'_{1235}. \qquad (4.9)$$

□

Remark 4.2 If $d'_2 = 0$, then both the intersection line of planes 1 and 2, and that of planes 2 and 3, are observed to be coincident after a motion (see Remark 3.2). On the other hand, if $d'_3 = 0$, then both the intersection line of planes 2 and 3, and that of planes 3 and 4, are observed to be coincident. These facts show that if $d'_2 \cdot d'_3 = 0$ then the number of visible lines changes before and after a motion. In this paper, we do not assume such a change occurs, which leads to $d'_2 \cdot d'_3 \neq 0$. □

5 Invariant of lines on two planes

In the previous section, we derived the properties characterized by the vectors that are obtained on the image plane when the values of the parameters ($\boldsymbol{n}_i$ and d_i), which uniquely determine lines, are given. Here, we investigate the properties of the vectors we can actually obtain, without assuming the known values of $\boldsymbol{n}_i$ and d_i.

5.1 Invariant of five lines

As stressed before, there is a scaling indeterminacy between vector $\boldsymbol{n}_{ij}$ that is derived from planes i, j (i, j are natural numbers), and the vector we actually obtain through observing their intersection line (see Remark 3.1). Hence, denote by $\boldsymbol{N}_{ij}$ the vector we actually obtain through observing the intersection line. Then

$$\boldsymbol{N}_{ij} \quad = \quad \rho_{ij}\boldsymbol{n}_{ij} \quad (\rho_{ij} \neq 0) \tag{5.1}$$

is satisfied. Here ρ_{ij} is a scaling factor and its value is not known. Define N_{ijkl} (i, j, k, l are natural numbers) as a counterpart of M_{ijkl}:

$$N_{ijkl} \quad := \quad [\, \boldsymbol{N}_{ij} \mid \boldsymbol{N}_{jk} \mid \boldsymbol{N}_{kl} \,]. \tag{5.2}$$

(5.1) and (5.2) yield

$$\det N_{ijkl} \quad = \quad \rho_{ij} \cdot \rho_{jk} \cdot \rho_{kl} \cdot \det M_{ijkl}. \tag{5.3}$$

We also denote by $\boldsymbol{N}'_{ij}$ the vector we actually obtain after a motion, and similarly define N'_{ijkl}. Note that $\boldsymbol{N}'_{ij} = \rho'_{ij}\boldsymbol{n}'_{ij}$ ($\rho'_{ij} \neq 0$) where the value of ρ'_{ij} is unknown. Then we obtain the following theorem.

Theorem 5.1 Let $\mathrm{rank}N_{i23j} = 3$ $(i = 1, 6\,;\, j = 4, 5)$. Then,

$$\mathrm{rank}N'_{i23j} = 3. \tag{5.4}$$

$$\frac{\det N_{1234} \cdot \det N_{6235}}{\det N_{1235} \cdot \det N_{6234}} = \frac{\det N'_{1234} \cdot \det N'_{6235}}{\det N'_{1235} \cdot \det N'_{6234}}. \tag{5.5}$$

Proof: Definition of N_{ijkl} leads to $\mathrm{rank}N_{ijkl} = \mathrm{rank}M_{ijkl}$. Similarly, $\mathrm{rank}N'_{ijkl} = \mathrm{rank}M'_{ijkl}$. These yield (5.4) from Lemma 4.1.

From (5.3) we obtain[3]

$$\begin{aligned}\text{LHS of (5.5)} &= \frac{\rho_{12}\,\rho_{23}\,\rho_{34}\;\rho_{62}\,\rho_{23}\,\rho_{35}\;\det M_{1234} \cdot \det M_{6235}}{\rho_{12}\,\rho_{23}\,\rho_{35}\;\rho_{62}\,\rho_{23}\,\rho_{34}\;\det M_{1235} \cdot \det M_{6234}} \\ &= \frac{\det M_{1234} \cdot \det M_{6235}}{\det M_{1235} \cdot \det M_{6234}}.\end{aligned} \tag{5.6}$$

On the other hand, it is easy to see

$$\begin{aligned}\text{RHS of (5.5)} &= \frac{\rho'_{12}\,\rho'_{23}\,\rho'_{34}\;\rho'_{62}\,\rho'_{23}\,\rho'_{35}\;\det M'_{1234} \cdot \det M'_{6235}}{\rho'_{12}\,\rho'_{23}\,\rho'_{35}\;\rho'_{62}\,\rho'_{23}\,\rho'_{34}\;\det M'_{1235} \cdot \det M'_{6234}} \\ &= \frac{\det M'_{1234} \cdot \det M'_{6235}}{\det M'_{1235} \cdot \det M'_{6234}}.\end{aligned} \tag{5.7}$$

By replacing plane 1 with plane 6 in (4.9), we obtain

$$\det M'_{6234}\,\det M_{6235} = \det M_{6234}\,\det M'_{6235}. \tag{5.8}$$

(5.6) and (5.7) yield (5.5) from (5.8) and (4.9) in Lemma 4.2. □

Theorem 5.1 shows that there exists a projective invariant

$$I := \frac{\det N_{1234} \cdot \det N_{6235}}{\det N_{1235} \cdot \det N_{6234}} \tag{5.9}$$

for five lines, which are characterized as the intersections of two of six planes.

In the subsequent sections, we characterize the structure of the above five lines and give a necessary and sufficient condition under which the invariant never becomes singular.

5.2 Construction of five lines

Projective invariant I is calculated from the following five lines:

[3] LHS and RHS mean the left-hand side and the right-hand side, respectively.

- L_{12} (the intersection line of planes 1 and 2),
- L_{23} (the intersection line of planes 2 and 3),
- L_{34} (the intersection line of planes 3 and 4),
- L_{35} (the intersection line of planes 3 and 5),
- L_{62} (the intersection line of planes 6 and 2).

The structure of these five lines in 3-D is characterized as follows:

1. The five lines are all on plane 2 or plane 3,
2. The intersection line of planes 2 and 3 is included (L_{23}),
3. There are two lines on plane 2 in addition to L_{23} (L_{12} and L_{62}),
4. There are two lines on plane 3 in addition to L_{23} (L_{34} and L_{35}).

Therefore, there exists a projective invariant I for five lines on two planes (planes 2 and 3 above). The five lines include the intersection line of the two planes, and two other lines on each plane.

5.3 Condition for nonsingularity

In this subsection, we give the necessary and sufficient condition under which the invariant I is nonsingular: the condition for nonsingularity of I. Here we define "An invariant is nonsingular" as "The value of the invariant is never any: 0, ∞, or $^0/_0$". From (5.9) it is easy to see that I is nonsingular if and only if the value of the determinant of N_{ijkl} is not zero. Therefore, the necessary and sufficient condition under which I is nonsingular is that the value of the determinant of M_{ijkl} is not zero (see (5.3)).

Observation 2.1 tells us that when three lines on the image plane do not share a common point, the value of the determinant of M_{ijkl} is never zero, which yields the following theorem (see Remark 4.1).

Theorem 5.2 The necessary and sufficient condition under which I in (5.9) is nonsingular is that the five lines on two planes have the following property:

For three lines, i.e., the intersection line of the two planes, and any two noncoplanar lines from among the other four, (I) and (II) are satisfied.

(I) They never share a common point in 3-D.

(II) They are not parallel. □

Since the size of an object we treat is finite, we can calculate the invariant I when we observe five line segments on two planar regions. Note that we have to observe the intersection line segment of the two regions. Therefore, the assumptions imposed on an object for which we can calculate the invariant I are 1) that there exist two adjacent planar regions, 2) that the intersection line segment of the two regions is observed, 3) that two other line segments on each region are observed, and 4) that these five line segments satisfy the condition for nonsingularity of the invariant (see Theorem 5.2).

Remark 5.1 When we observe six points on two planes such that

- there are two points on the intersection line of the two planes and,
- there are two other points on each plane,

it is easy to see that we can construct five lines on two planes, from which we can calculate invariant I and which satisfy the condition for nonsingularity (see Fig. 3). This shows that there exists the same invariant for six points on two planes. □

6 Experimental results

In Section 5, we proved the existence of the projective invariant I of five lines on two adjacent planar regions. The five lines there include 1) the intersection line of the two regions; and 2) two other lines on each region. In addition, to guarantee nonsingularity of the invariant, two conditions have to be satisfied: 3) that the intersection line and any two noncoplanar lines from among the other four never share a common point in 3-D; and 4) that the intersection line and any two noncoplanar lines from among the other four are not parallel. On the basis of these results, our experimental results with real images are shown.

We obtained several images[4] of the polygon in Fig. 4 using a fixed camera. The polygon was randomly moved by hand. For each image, we first applied a low pass filter of a 3×3 weighted kernel window to reduce noise. We then calculated the Laplacian with an 8-neighbor weighted coefficient matrix to extract the edges (Fig. 6). Next, to each edge in the image, we apply the method of least squares to find the equation of the line that represents the edge. On the other hand, we attached labels to the planar regions and edges of the polygon (see Fig. 5) and chose the planar regions (A) and (B) as two adjacent. We then selected five of seven edges $1, 2, \ldots, 7$ on either planar region (A) or (B) to calculate the value of invariant I. There are nine combinations of five of the seven lines that include[5] line 3, the intersection of planar regions (A) and (B), and two other lines on each of planar regions (A) and (B). However, we essentially have only four combinations that give independent values of the invariant (see definition I in (5.9)). Thus, for the lines that were obtained from six edge images (a), ..., (f) in Fig. 6, we calculated the values of these four invariants, which are shown in Table 1. We denote by I_{ijklm} the invariant of five lines i, j, k, l, m ($i, j, k, l, m \in \{1, 2, \ldots, 7\}$). For each invariant, we also showed the mean m over the six images, the standard deviation σ and the percentage of the standard deviation of the mean.

Table 1 shows that all of the values of I_{ijklm} are almost constants: they remain stable in spite of a change in viewpoint. Furthermore, their values significantly depend on a choice of five lines, i.e., a combination of observed lines, which shows that each object generally has its own proper value of I. Therefore, the value of I can be important in identifying one object out of many.

As shown above, for a real 3-D object we found the invariant I that is unaffected by a change in viewpoint and which has its own proper value.

7 Conclusion

We proved the existence of one projective invariant I of five lines on two adjacent planar regions. These five lines include 1) the intersection line of the two regions; and 2) two other

[4]Each image consists of 480×512 pixels. And each pixel is assigned a natural number from $0 \sim 255$ as the value of its grey level.

[5]The line i denotes the line representing the edge i ($i \in \{1, 2, \ldots, 7\}$).

lines on each region. Furthermore, the necessary and sufficient conditions for nonsingularity of the invariant are 3) that the intersection line and any two noncoplanar lines from among the other four never share a common point in 3-D; and 4) that the intersection line and any two noncoplanar lines from among the other four are not parallel.

We applied these theoretical results to real images, and found that the values of the invariant remain stable even for noisy images. Furthermore, we also found that an object generally has its own proper value of the invariant. Left for future investigation is the theoretical analysis of the noise sensitivity of the invariant.

Even though the importance of projective invariants to a number of machine vision tasks has been recognized for many years, we have found quite a few invariants for 3-D objects. When we impose no assumption on an object, we cannot obtain any invariant from a single view [5], [6]. However, if we impose some assumptions on an object, an invariant can be derived from a single view as shown in this paper. We should concentrate on how far we can generalize our current results so that we can use invariants with greater frequency. It should also be helpful to investigate properties for the case where an invariant becomes singular so that we can use invariants even in that case.

Acknowledgments

The author is grateful to Yoh'ichi Tohkura and Shigeru Akamatsu of ATR Human Information Processing Research Laboratories for providing the opportunity for this research. He also appreciates Shinjiro Kawato there for providing access to his system to obtain real images. He deeply thanks Kazuo Murota of Research Institute for Mathematical Science of Kyoto University for comments on an earlier version of this paper.

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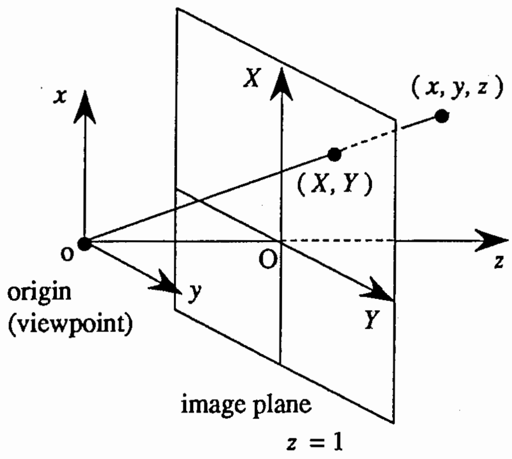

Fig. 1: Perspective projection centered at the origin

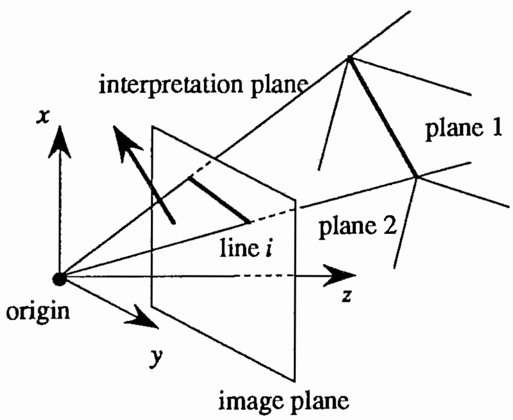

Fig. 2: Line i and the normal vector of its interpretation plane

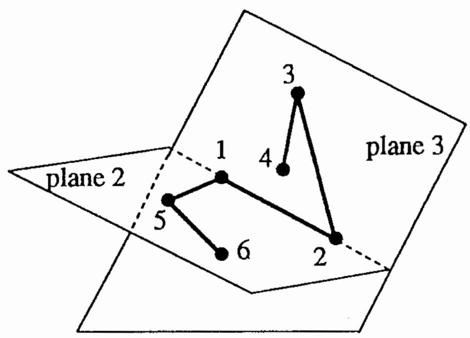

Fig. 3: Five lines derived from six points on two planes

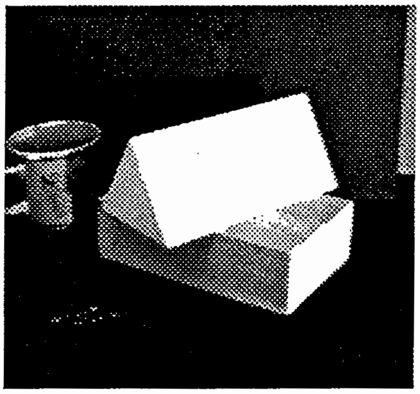

Fig. 4: A polygon used to calculate projective invariant

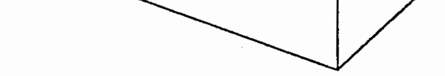

Fig. 5: Labels for planes and lines of the polygon in Fig. 4

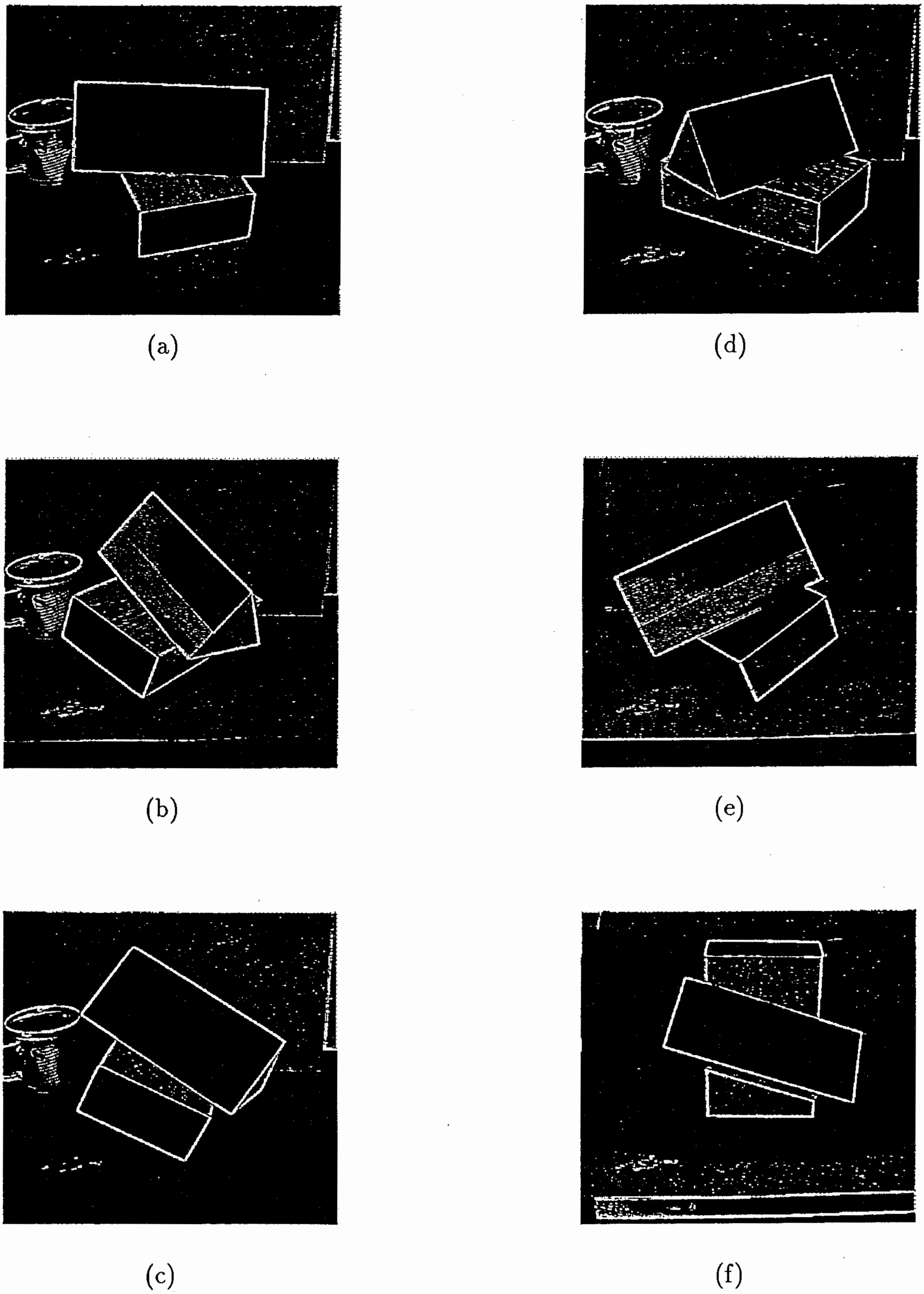

Fig. 6: Extracted edges from images of the polygon in Fig. 4

We obtained several images of the polygon in Fig. 4 using a fixed camera. The polygon was moved randomly by hand. For each image, we first applied a low pass filter of a 3×3 weighted kernel window to reduce noise. We then calculated the Laplacian with an 8-neighbor weighted coefficient matrix to extract edges ((d) is the extracted edge image for Fig. 4).

Table 1: Values of invariant I

Shown are the values of I calculated from the lines representing the edges in Fig. 6. Also shown are their means m, standard deviations σ and the percentages of the standard deviations of the means.

	I_{24357}	I_{24367}	I_{14357}	I_{14367}
(a)	0.043451	−0.46719	0.15229	−0.30025
(b)	0.039659	−0.39997	0.14043	−0.25306
(c)	0.042730	−0.45133	0.14967	−0.28919
(d)	0.043441	−0.43375	0.15562	−0.26561
(e)	0.039769	−0.39784	0.14500	−0.24465
(f)	0.041425	−0.46856	0.14702	−0.30678
m	0.041746	−0.43644	0.14834	−0.27659
σ	0.001583	0.02894	0.004880	0.02354
σ/m (%)	3.79	6.63	3.29	8.51