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# Feedforward Neural Network Modeling of Target-directed Arm Movement which Reproduces Speed-Accuracy Trade-off<sup>1</sup>

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#### Abstract

Various properties of the cascade neural network as a computational model for motor control of a multi-joint arm are studied. The cascade neural-network model calculates the trajectory based on minimum-torquechange criterion. If the weighting parameter of the smoothness criterion is fixed and the number of relaxation iterations is rather small, the cascade model cannot calculate the exact torque, and the hand does not reach the desired target using the feedforward control alone. Thus, one observes an error between the final position and the desired target location. By simulating target-directed arm movements using a fixed weighting parameter value and a limited iteration number, we found the cascade model reproduced the planning time-accuracy trade-off, and speed-accuracy trade-off of the arm movement, well known as Fitts's law. This work provides a candidate of possible neural mechanism which explains the stochastic variability of the time course of the feedforward motor command along with several invariant features of multi-joint arm trajectories such as roughly straight hand paths and bell shaped speed profiles.

When one plans and controls a motion, what is executed by the brain? To plan and control a voluntary movement, the brain must determine a single trajectory from an infinite number of possible trajectories and activate muscles to realize the desired trajectory. How is this done?

In case of a target directed voluntary arm movement, when a hand is moved to a target, the central nervous system must select one specific trajectory among an infinite number of possible trajectories that lead to the target position. Several researchers measured human arm movements and found some invariant features.

One beautiful feature of human multi-joint arm movements is that hand paths between two points are roughly straight, and hand-speed profiles are bell-shaped (Kelso, Southard, & Goodman, 1979; Morasso, 1981; Abend, Bizzi, & Morasso, 1982; Atkeson & Hollerbach, 1985; Flash & Hogan, 1985; Uno, Kawato, & Suzuki, 1989). To account for such features, Flash and Hogan (1985) proposed that the trajectory followed by human subject arms tended to minimize the Cartesian jerk of the hand. This minimum jerk model reproduced the qualitative features and the quantitative details of the human hand trajectories between two targets which are located approximately in front of body. Uno, Kawato and Suzuki (1989) studied this model further by considering optimization of dynamical quantities, and proposed the minimum torque-change model. This model

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is formulated by defining an objective function, a measure of performance for any possible movement: the square of the rate of change of torque integrated over the entire movement. That is, the objective function  $C_T$  is defined as follows:

$$C_T = \frac{1}{2} \int_0^{t_f} \sum_{i=1}^n \left(\frac{d\tau_i}{dt}\right)^2 dt$$

where  $\tau_i$  is the torque generated by the *i* th actuator (muscle) out of *n* actuators, and *t<sub>f</sub>* is the movement time. Uno et al. experimentally examined human arm movements. Trajectories and speed profiles of human arm movements were predicted quite well by the minimum torque-change criterion, in several behavioral situations including horizontal point-to-point movements, via-point movements, constrained movements in which a spring force acts on the hand, and vertical point-to-point movements.

We proposed a cascade neural network model for trajectory formation based on the minimum torque-change criterion (Maeda, Kawato, Uno, Suzuki, 1989, Kawato, Maeda, Uno, Suzuki, 1990). In this paper, we further study various properties of the cascade neural network model as a computational model for motor control of a multi-joint arm. We implemented a computer simulation program of the cascade neural network model and examined its properties in trajectory formation. Neural network models which reproduce speed-accuracy tradeoff have been proposed based on internal or external feedback loops (Bullock and Grossberg, 1988; Hoff and Arbib, personal communication). We will show that the cascade neural network model can reproduce the above mentioned invariant features of multi-joint arm trajectories as well as the speed-accuracy tradeoff based on feedforward control.

# **Cascade Neural Network Model**

Figure 1 shows the cascade neural network model for trajectory formation based on the minimum-torquechange criterion (Maeda et al., 1989, Kawato et al., 1990).

The basic schemes of information representation and algorithms are (i) spatial representation of time, (ii) learning of forward dynamics and kinematics model and (iii) relaxation computation based on the acquired model. The operation of the cascade neural network model is divided into learning and trajectory formation phases.

In the learning phase, the network acquires a forward model of the multi-degrees-of-freedom controlled object while monitoring the actual trajectory as a teaching signal. In the trajectory formation phase, electrical coupling between neurons representing sequential motor commands is activated to guarantee the minimum torque-change criterion. The network changes its state autonomously by forward calculation through the cascade structure, and by error back-propagation based on the acquired model.

As shown in Figure 1, the cascade model consists of many identical four-layer network units. The j th network unit corresponds to time  $j\Delta t$ . The network units are connected in a cascade formation. The 1st layer of the network unit represents the time course of the trajectory and the torque. The 3rd layer represents the trajectory change per unit of time. The 4th layer and the output line on the right side represent the estimated time course of the trajectory. In short, the three layer part of each network unit calculates the vector field of a dynamical system which describes a controlled object, and hence the cascade network provides a hardware implementation of Euler's method of numerical integration of differential equations. The number N of cascade units is related to movement time. The cascade model can generate trajectories of any movement time shorter than  $N\Delta t$ .

In the trajectory formation phase, electrical couplings between neurons representing torque in the 1st layer of the neighboring units are activated. The electrical couplings are designated as electrical resistance in Figure 1. The electrical conductance guarantees smoothness of the motor command, that is, a minimization of rate of change of torque.

The central commands specify the desired target position, the desired via-points and the locations of obstacles to be avoided, and give this information to the 4th layer of the cascade network. These positions are represented in task-oriented coordinates such as Cartesian coordinates of the hand position. The hand trajectory is first estimated by the feedforward calculation of the cascade network based on the current values of torque represented by the motor command neurons in the 1st layer. Then, errors between the desired target position or the desired via-points and the estimated trajectory are calculated at the output line on the right side of the cascade network. Next, these errors are backpropagated all through the cascade structure to motor command neurons in the 1st layer. Finally, the state of the motor command neurons is updated according to summation of two forces: A smoothing force due to electrical conductance, and an error correcting force due to backpropagation calculation.

It can be shown mathematically that the cascade network executes the steepest descent motion with respect to

the following energy which consists of two terms.

$$E = E_D + gE_S = E_D + g \int_0^{t_f} \sum_k \left(\frac{d\tau_k}{dt}\right)^2 dt$$

where  $\tau_k$  is the torque generated by the k th actuator (muscle) and tf is the movement time.  $E_D$  corresponds to the hard constraint which requires that the hand reaches the target, and passes through the via-points. The smoothness energy  $E_S$  is the objective function of the minimum torque-change model. The smoothness constraint is weighted by a positive value g, which corresponds to the electrical conductance of gap junctions in the cascade network. Relaxation is conducted by backpropagating the positional error of  $E_D$  through the cascade structure. Mathematically, the forward calculation through the cascade structure corresponds to the forward integration of the dynamical system of the controlled object. Correspondingly, backpropagation through the cascade structure is related to backward integration of the adjoint equation of the dynamical system. The cascade network as a whole relaxes the energy by repetition of forward and backward calculations. The number of iterations for relaxation calculation corresponds to the planning time of the movement.



Figure 1. Cascade neural network model.

# Simulation Method

We implemented the cascade network simulation program and simulated trajectory formation of target directed arm movements with various durations of movement. Simulation configurations are described as follows.

## Arm Model

A two-joint manipulator with two-degrees-of-freedom is used as a model of a human right arm. Figure 2 shows the model and target locations used for the simulation. Physical parameters of the model manipulator were chosen based on experiment data and human arm geometry (see Uno et al., 1989).



Figure 2. Arm model and target locations.

## **Arm Dynamics**

In the previous studies, we confirmed that the forward dynamics of the arm can be acquired in the cascade neural network model by the usual backpropagation learning algorithm (Maeda et al., 1989, Kawato et al., 1990). In this paper, we are mainly interested in the trajectory formation capabilities of the model. Thus, we used an exact dynamics equation instead of a multi-layer feedforward network unit. The following equations are used for a two-joint manipulator within the plane.

$$\tau_{1} = (M_{2}L_{1}^{2} + 2M_{2}L_{1}S_{2}\cos\theta_{2} + I_{1} + I_{2})\ddot{\theta}_{1} + (M_{2}L_{1}S_{2}\cos\theta_{2} + I_{2})\ddot{\theta}_{2}$$
$$- M_{2}L_{1}S_{2}(2\dot{\theta}_{1} + \dot{\theta}_{2})\dot{\theta}_{2}\sin\theta_{2} + B_{1}\dot{\theta}_{1}$$
$$\tau_{2} = (M_{2}L_{2}S_{2}\cos\theta_{2} + I_{2})\ddot{\theta}_{1} + I_{2}\ddot{\theta}_{2} + M_{2}L_{2}S_{2}\dot{\theta}_{1}^{2}\sin\theta_{2} + B_{2}\dot{\theta}_{2}$$

## Arm Kinematics

The higher motor center in Figure 1 gives information about locations of the target point, via-points and obstacles to the cascade model in task-oriented coordinates; the Cartesian coordinate of the hand position in this case. Hence, the cascade network needs to solve the so called inverse kinematics problem to determine joint angles from the corresponding hand positions. There are two different ways to resolve this problem (see Kawato

#### et al., 1990).

The first is to represent the combination of both the forward dynamics and forward kinematics using the 3layer network unit in Figure 1. That is, the dynamics of the arm are described by using both the joint angle and hand position variables.

The second approach is to divide the forward dynamics model and the forward kinematics model. In this case, the cascade structure itself represents only forward dynamics. We separately prepare the forward kinematics model which calculates the hand position from the joint angles, and is attached to the output layer of each network unit. Thus, N identical forward kinematics model networks are necessary because there are N network units in the cascade model. We found that both of the above two schemes worked well (Maeda et al., 1989, Kawato et al., 1990).

In this paper, we adopt the second scheme since it is more suitable to utilization of exact equations. Figure 3 shows the simulation model of the cascade neural network model using F.D.M.(Forward Dynamics Model) and F.K.M.(Forward Kinematics Model).

Hand position errors are first calculated at the output line of the forward kinematics model. These errors are then backpropagated through the forward kinematics model to calculate errors in joint angle space. This procedure calculates the joint angle error  $e_J$  as the product of the transpose of Jacobian of coordinate transformation  $J^T$  and the hand position error  $e_T$  (see for example Jordan, 1991):

$$e_{I} = J^{T} e_{T}$$



Figure 3. Cascade neural network model with F.D.M. (Forward Dynamics Model) and F.K.M. (Forward Kinematics Model).

#### **Trajectory Formation with Fixed Electrical Conductance**

The cascade network executes the steepest descent motion with respect to the weighted sum of the smoothness constraint and the hard constraint regarding target points. The value of the electrical conductance is the weight of the smoothness term. The electrical conductance must be slowly decreased to zero so that the hard constraint is strictly satisfied, well known as the "penalty method" in optimal control theory.

In our previous simulation experiments, the electrical conductance was slowly decreased to zero (Maeda et al., 1989, Kawato et al., 1990). However, this "simulated annealing" procedure requires a vast number of iterations, and is highly biologically implausible because the central nervous system must calculate the feedforward torque

within a few hundred milliseconds. We resolved this difficulty in the cascade model from three different viewpoints. The first two are efforts to reduce the required number of iterations. The third, the main topic of this paper, is to analyze the nature of the errors induced by the fixed electrical conductance and a small number of iterations.

We first briefly illustrate the first two approaches. Kitano, Kawato, Uno, Suzuki (1990) developed a novel method called the "virtual target point" to reduce the number of iterations even with a fixed and relatively large value of the electrical conductance. The virtual target point moves around to compensate for the effect of the fixed electrical conductance. We can mathematically show that this generates a rigorous minimum torque-change trajectory. Kitano found that a quite good trajectory for a two-joint arm within a plane can be calculated with only 80 iterations (Kitano et al., 1990). The second approach is to use a good initial torque waveform for relaxation computation. In all simulations, we chose the zero initial torque waveform which is, of course, a poor choice. We can imagine an associative content addressable memory (ACAM) neural network which can store the equilibrium solution calculated by the cascade network, and can instantaneously load it on the cascade network as a good starting point for relaxation computation. One potential ACAM candidate is Jordan's recurrent network which can calculate a smooth trajectory in real time (Jordan, 1990).

In this paper, however, we do not use either of the two improvements. Instead, we use a fixed electrical conductance and zero initial condition for the torque waveform. Thus, we can expect that the number of iterations reported in this paper could probably be reduced by one or two orders of magnitude if the two improvements were utilized.

#### Time and Accuracy of Target-directed Movements

Target-directed voluntary movements can be thought of as a series of processes, which are (1) inputs of visual information such as a start position, a target position and environmental situations, (2) planning of motor command according to some constraints, (3) activation of muscles by feeding the generated motor command. Consideration of the timing of each process is helpful in elucidating the motor control mechanism. Today, human motions were measured and analyzed by several researchers with these results sometimes being used to predict human performance in industry or to design user interfaces of machines.

Fitts measured human subjects' movements and found a speed and accuracy trade-off. The relationship of speed and accuracy is expressed by the following equation known as Fitts' law (Fitts, 1954, Fitts & Peterson, 1964).

$$MT = a + b \log_2\left(\frac{2A}{W}\right)$$

where MT is movement time, A is amplitude of movement, W is width of a target, a and b are constants which depend on behavioral situations.

We simulated the cascade neural network for movements of various durations. This can be done by changing the number of network units to which the target position is given.

# Result

#### Trajectories

Figure 4 shows results of computer simulations with the fixed electrical conductance of g = 0.001 with fixed iteration number of 2500 for various arm movements with 0.7 second duration. We show hand paths, hand tangential velocities, which were realized and produced by the cascade neural network. Five point-to-point trajectories (T1-T3, T2-T6, T3-T6, T4-T6, T4-T1) were generated. The start and target points are the same as those of the human behavioral experiments studied by Uno et al. (1989). Trajectories and velocities produced by the cascade neural network were in accordance with the data of human movements, even though the conductance of the electrical synapse was fixed.



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a) Hand paths of 5 target directed movements (T1-T3, T2-T6, T3-T6, T4-T1 and T4-T6)in 0.7s movement time.



Figure 4. Trajectories produced by the cascade neural network model.

## **Planning Time-Accuracy Trade-off**

Figure 5 shows the final position error of the hand as a function of the iteration number of the relaxation for the movement T2-T6. A duration of the movement was either 0.5, 0.6, 0.7, 0.8, 0.9 or 1.0 second. The error to the desired target position is defined as the Euclidean distance between the desired target point and the end point of the trajectory which was calculated by the cascade neural network in 2500 iterations of relaxation. Because we used the exact model of the forward dynamics and forward kinematics in the cascade model, the estimated trajectory is almost identical to the realized trajectory of the manipulator controlled only by the feedforward torque calculated by the cascade network. Consequently, the error can be regarded as the actual position error at the end of movement. The error decreased with the number of iterations, that is, with the planning time of movement. This is because the iteration number for relaxation calculation can be regarded as the planning time. This result shows the planning time and accuracy trade-off.

Furthermore, the error is larger for shorter duration. This suggests speed-accuracy trade-off. The final position error did not approach zero even with a large number of iterations because of the fixed electrical conductance.



Figure 5. Planning time-Accuracy trade-off of a target directed movements (T2-T6) in various movement time (0.5, 0.6, 0.7, 0.8, 0.9 and 1.0s), produced by the cascade neural network model.

## Speed-Accuracy Trade-off

Six movement durations (0.5, 0.6, 0.7, 0.8, 0.9, 1.0 sec.) were examined for 10 different trajectories (T1-T3, T2-T6, T3-T6, T4-T1, T4-T6 and T2-T6, T2a-T6, T2b-T6, T2c-T6, T2d-T6). The initial and final target points are shown in Figure 2. T1-T3, T2-T6, T3-T6, T4-T1, T4-T6 are movements with different directions and different amplitudes. As can be seen in Figure 2, T2-T6, T2a-T6, T2b-T6, T2c-T6, T2d-T6 share the same direction, and the latter four movements are parts of the first full movement. The average time of human subject movements was approximately 0.7 second in our experiments for T1-T3, T2-T6, T3-T6, T4-T1 and T4-T6 trajectories (Uno et al., 1989).

Figure 6 shows the final position error as a function of speed 1/MT and as a function of amplitude of movement. The final position error is called the effective width We and can be regarded as the width of the target in a Fitts' type behavioral experiment. T2-T6, T2a-T6, T2b-T6, T2c-T6, T2d-T6 trajectories are used for this plot. This result shows that for the same amplitude of movement, the error is larger for the larger speed of movement (Figure 6a). For the same duration of movement, the error is larger for the larger amplitude of movement (Figure 6b). In addition, the relationship between the amplitude of movement and the error is almost linear for the same duration of movement. If Fitts' law holds, 2A / We is constant for the same MT. Simulation results shown in Figure 6 obey this rule.





The accuracy  $\log_2\left(\frac{2A}{We}\right)$  which was defined by Fitts was calculated for five different trajectories; T1-T3, T2-T6, T3-T6, T4-T1, T4-T6. Figure 7 shows the movement time as a function of this accuracy. The result shows that movement time and accuracy trade-off is in accordance with the Fitts' law. That is, the cascade

neural network model predicts speed-accuracy trade-off of human arm movements. One may notice that the straight lines for 5 different trajectories in Figure 7 are different although the movement time as a function of accuracy obeys Fitts' law. That is, constants a and b depend on movements. Table 1 shows the constants a and b calculated from this graph. This prediction that the speed at which a movement proceeds is based not only on the accuracy and distance requirements of the task but also on biomechanical factors, such as, the summation of interactional forces was recently confirmed by experimental data (Bassile & Kaminski, 1991).

Our simulation held movement time constant at each of several values and then measured the variability of movement endpoint. In the classification of Meyer, Smith Kornblum, Abrams, and Wright (1990) this is a time-matching movement task. Movement made under this set of task constraints typically exhibit a linear speed-accuracy trade-off  $We = K_1 + K_2(A/MT)$  (see Wright and Meyer, 1983 for experimental support). Fitts' law, in contrast, is usually observed in situations that were classified in that paper as time-minimization movement task. In this class of movements the goal of the subject is to move as fast as possible given the constraints on the movement endpoint. Thus if this classification is applied also to multi-joint arm movement with relatively long movement durations, our simulation result should obey the linear speed-accuracy trade-off. The simulation data was approximately described by the linear law for a small range of velocity variation. However, for the total range simulated, the log law accounts better our simulation result than the linear law.

If the linear law holds, we should see family of straight lines in Fig. 6a which plots effective target size as a function of speed, but the simulated curves have marked concavity. It might be interesting to observe similar concavity in the experimental data for the time matching task (for example Figure 1 of Zelaznik et al., 1988). We have recently proposed a new neural network model for minimum-torque-change trajectory (Kawato, 1992). It is interesting to examine whether the new model can reproduce linear speed-accuracy trade-off. Examination of single-degree-of-freedom controlled objects such as wrist or eye with the cascade neural-network model is also interesting.





Trajectory	a (ms)	b (ms)
T1 - T3	-125	113
T2 - T6	-91	111
T3 - T6	-61	110
T4 - T1	-124	106
T4 - T6	-34	109

Table 1. Constants a and b for the Fitts' law equation

# Discussion

The classical explanation of Fitts' law invokes feedback corrections at long intervals (see, for example, Keele, 1986). We think this explanation fails if one considers a relatively long feedback delay. The loop time, which includes sensory processing by photoreceptors in the retina, planning and motor-command generation, and activation of muscles, may exceed 100 msec (Evarts, 1974). If one has experience with conventional real-time feedback control (for example using usual PID controllers), it is evident that control of a 700-msec movement with 100-msec feedback delay is very difficult. However, it must be noted that a feedback control with 100-msec delay could contribute fine in slow movements or in posture control. Bizzi et al. (1984) reported data which supports that a desired movement trajectory is explicitly planned in the brain and then feedforward control executes it. In their experiments, when the forearm of a deafferented monkey or an intact monkey was quickly forced to a final target position early in a movement, the arm returned to some intermediate point between the initial and final positions, then gradually approached the final target position again.

Even an elegant and comprehensive theory, the "stochastic optimized-submovement model" proposed by Meyer et al.(1990), relies on a feedback signal for starting a secondary corrective submovement in order to hit a target. If one assumes that the error signal for corrective submovements requires detecting the final position of the first submovement, there must exist at least 50 msec dead time due to somatosensory feedback before the second submovement. This is not the case because the typical oscillation of movement velocity and acceleration around the end of the first ballistic movement is continuous (Meyer, Abrams, Kornblum, Wright and Smith, 1988). Thus, Meyer et al. (1988) assumed that feedback and feedforward (efference copy) are processed "on the fly" during movement production.

In our opinion, the submovements reported by Meyer et al. (1990) instead might be interpreted as physical oscillation caused by visco-elastic properties of the musculoskeletal system. It might be hypothesized that the initial ballistic part of a movement is controlled by a strictly feedforward mechanism like the cascade neural network, whereas the later part of the movement is executed by a posture controller that specifies levels of stationary motor commands for groups of muscles based on visual information about target location. Presumably, the use of visual feedback is stationary. If the final position of a hand realized by the feedforward control coincides with the stationary posture commanded by the feedback mechanism (posture controller), there happens no oscillation. However, when these two positions are different, then damped oscillation which converges to the specified posture should be observed because of the spring-like dynamics of the musculoskeletal system in combination with the posture controller. Thus, this mechanism explains why and how this "passive" oscillation made the movements more accurate when there was visual feedback. Furthermore, when concurrent visual feedback of the current position is removed, the posture controller suffers from inaccurate coordinate transformation between the target visual location and necessary motor commands because of loss of the reference. Then, under this condition, our model predicts inaccurate final positioning and decrease of stiffness of the final posture (see Meyer et al., 1988 for related experimental results). We proposed one candidate for this posture controller: the inverse-statics model (Katayama & Kawato, 1991).

Nevertheless, we agree the view that variability in motor-output processes mediates errors in rapid movements, which is the basic assumption of the stochastic optimized-submovement model (Meyer et al., 1990). This viewpoint was originally proposed in an impulse-variability model, and validated by behavioral experiments on controlling ballistic force pulses (Schmidt, Sherwood, Zelaznik, & Leikind, 1985). Our own subsequent work provides one possible neural mechanism that explains the stochastic variability in the time course of feedforward motor commands. From simulations reported in this paper and by Uno and Suzuki (1990), we infer that the calculated feedforward torque contains stochastic variability associated with variability in the number of iterations during relaxation computation, and with variability in electrical resistance values, or with variability in the learned forward model (Uno & Suzuki, 1990). Furthermore, the cascade neural-network model explains this variability for movements involving multiple degrees of freedom and a controlled object with realistic dynamics. These dynamics contain centripetal and Coriolis forces and frictional forces. The presence of such realistic forces violates basic assumptions of Meyer, Smith, & Wright (1982) about force-time

rescalability and symmetry.

Results shown in Figure 7 tempted us to invoke a totally feedforward mechanism which gives Fitts's law. However, we do not intend to totally deny the role of the feedback loop in movement. We are concerned about the concavity of the Fitts's law functions in Figure 7. If some visual feedback is utilized for the long movements, it might be enough to pull the concavity down. We hypothesize that the initial ballistic part of the movement is controlled by an entirely feedforward mechanism like the cascade neural network, and the later oscillatory part is explained by spring-like properties of the musculoskeletal system combined with a posture controller which specifies levels of stationary motor commands for groups of muscles based on the sensory feedback information about the target location. However, we emphasize that the use of feedback information is stationary such as by the inverse statics model.

In summary, the movement speed-accuracy trade-off may be explained by difficulty in feedforward neural calculation of ballistic motor commands for a controlled object with multiple degrees-of-freedom. Our present study is less refined in several ways (especially treatment of "time-matching" versus "time-minimizing") compared with some others (for example, the elegant and comprehensive model proposed by Meyer et al., 1990). Nevertheless, for the first time, we have tried to explain motor-command variability based on a specific, neural model of motor-command generation. Furthermore, our model extends previous theories about movements with a single degree of freedom and oversimplified dynamics to coordinated movements with realistic dynamics. Perhaps the strongest virtue of the cascade neural-network model is that it can reproduce both quantitative features of multi-joint movements and motor-command variability.

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